Layered Area-Proportional Rectangle Contact Representations

Martin Nöllenburg, Anaïs Villedieu, Jules Wulms

16.9.2021 · GD 2021

Funded by the Austrian Science Fund (FWF) under grant P31119

Background

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[Viegas et al. 2009]

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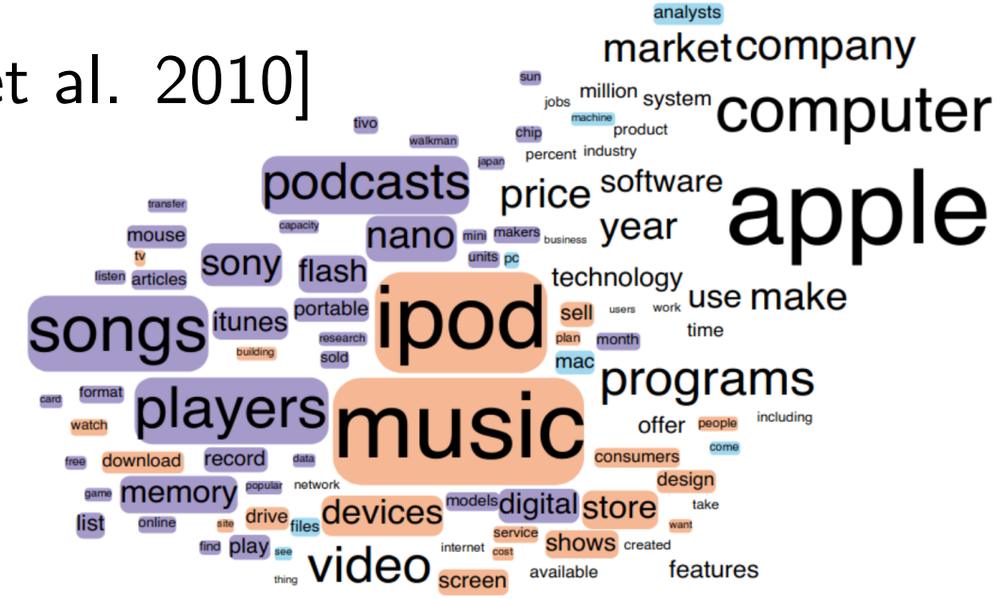
[Cui et al. 2010]



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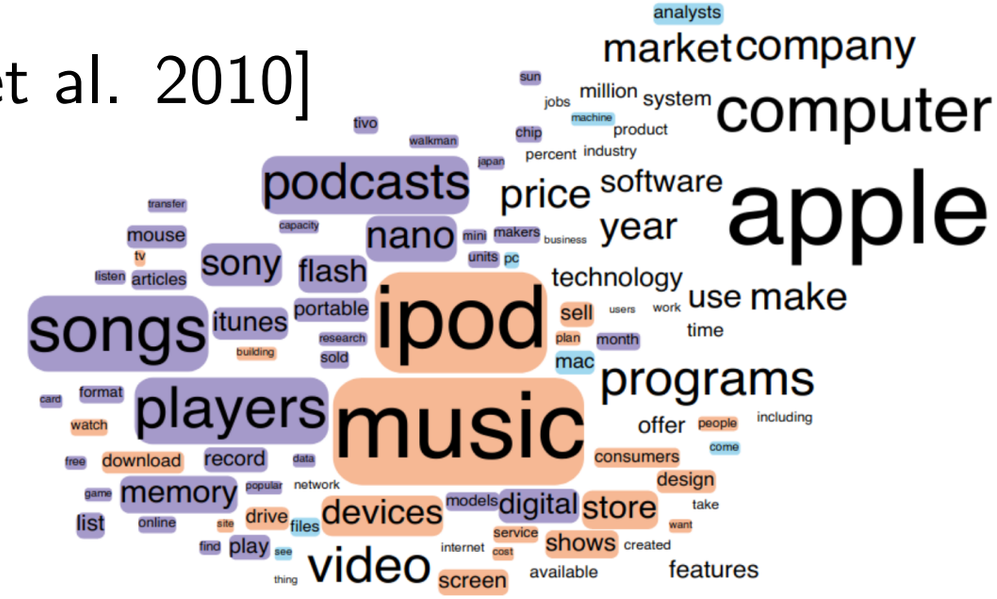

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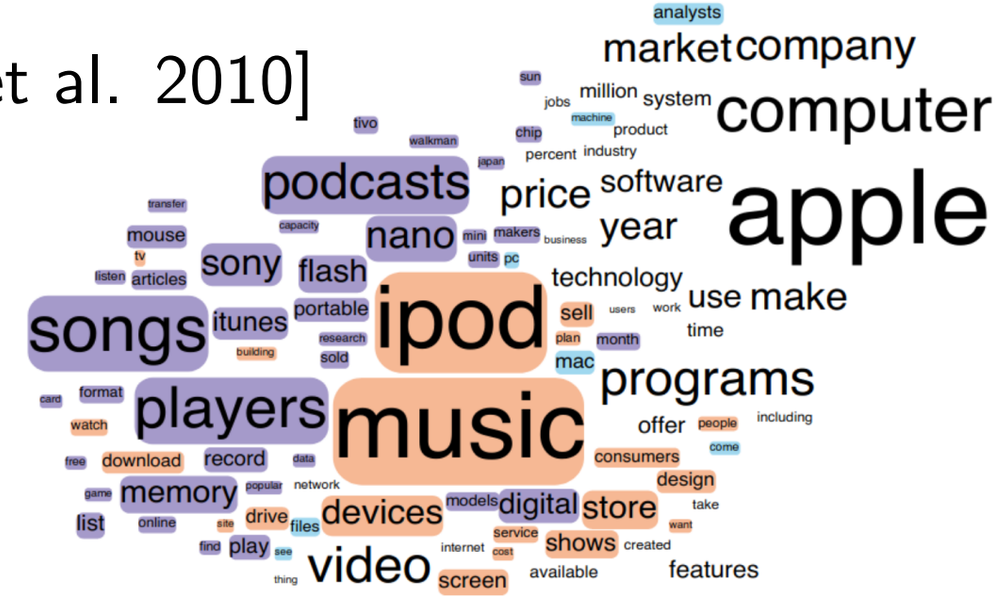

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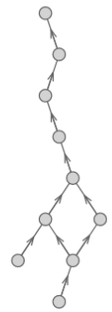

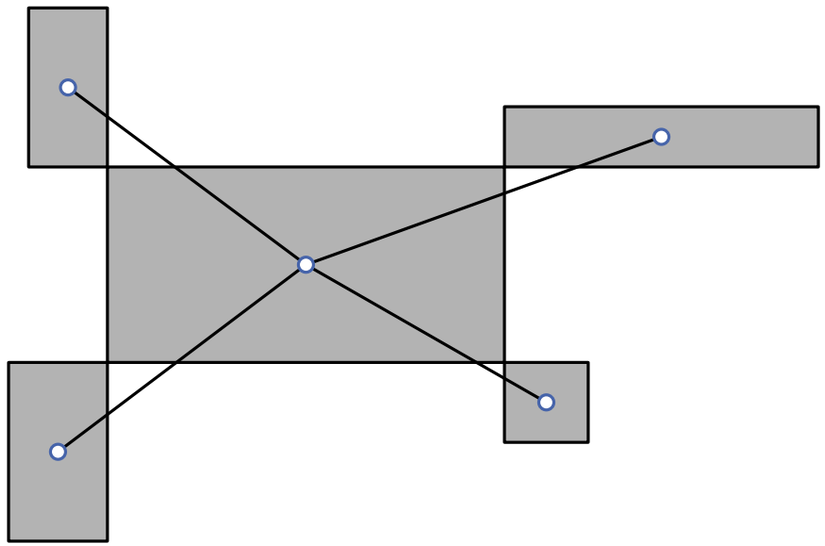

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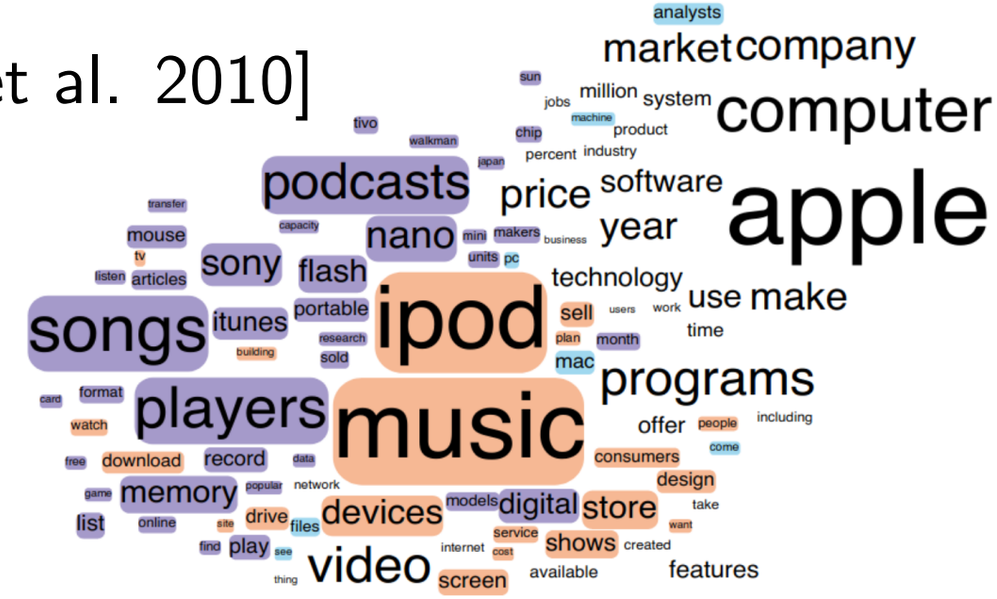

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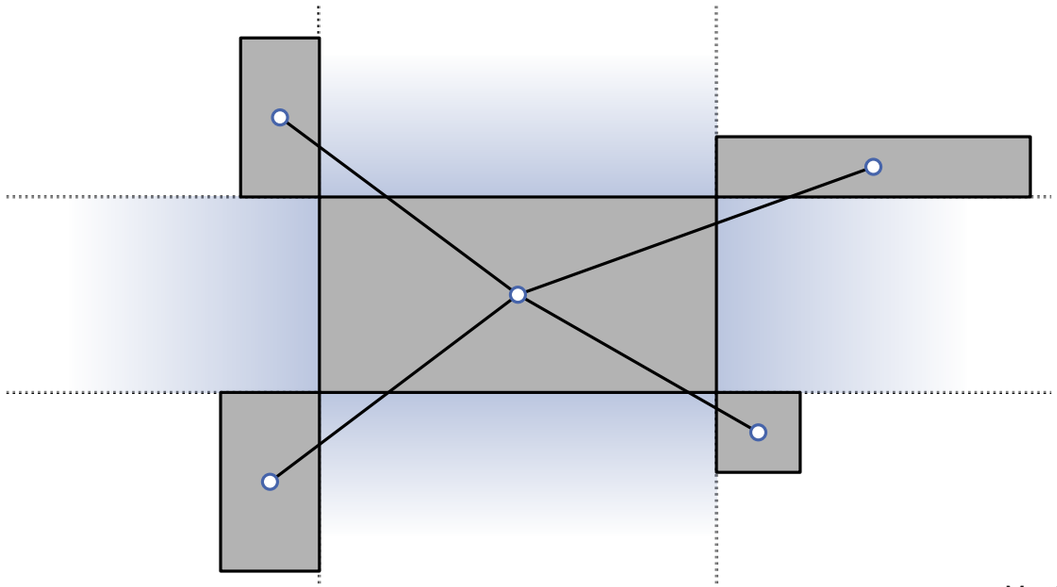

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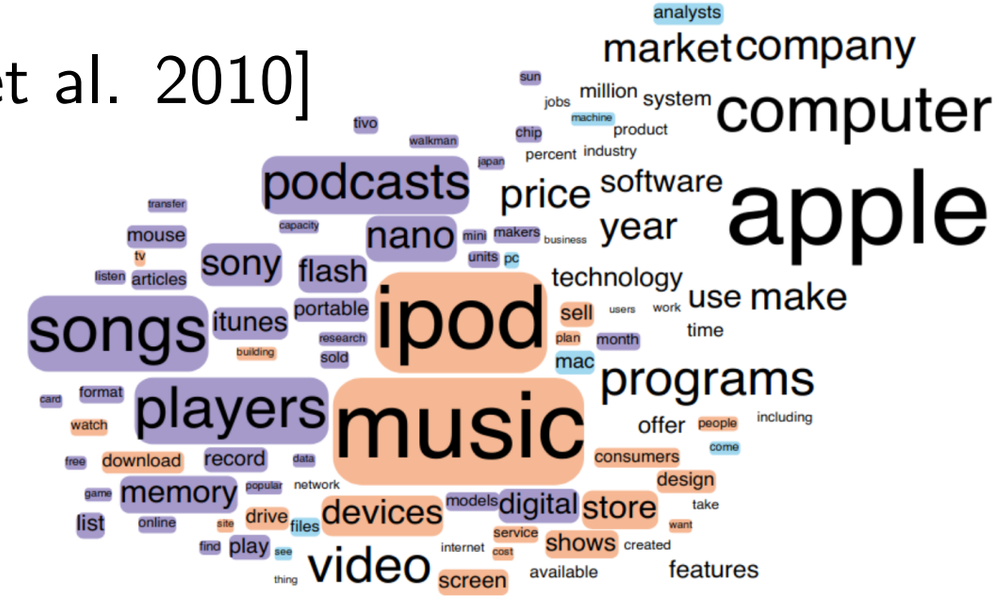

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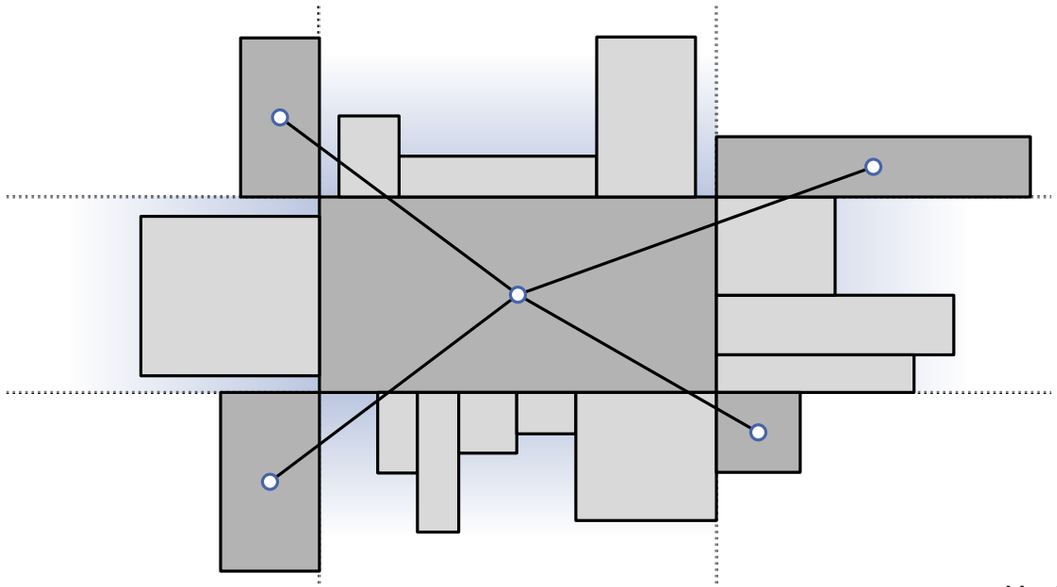

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Related works

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CROWN problem (Contact Representation of Word Networks)

Maximize $p = \sum_{e \in E_r} w_e$, where E_r are realised edge contacts

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$\rightarrow$ based on generalized assignment problem approximation and graph decompositions

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- Area proportional
  - Fixed aspect ratio
  - Unit height

  $\rightarrow$ fixed width

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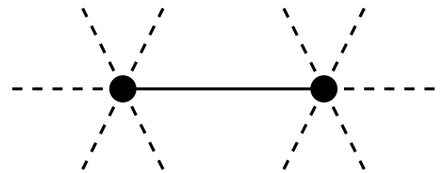

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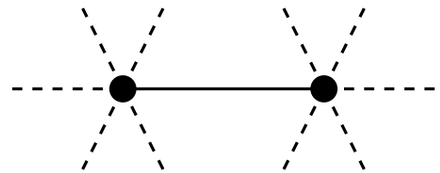

wordle representations

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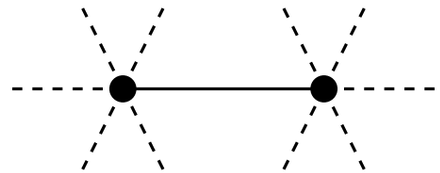

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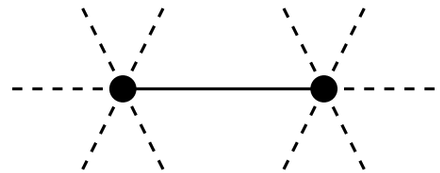

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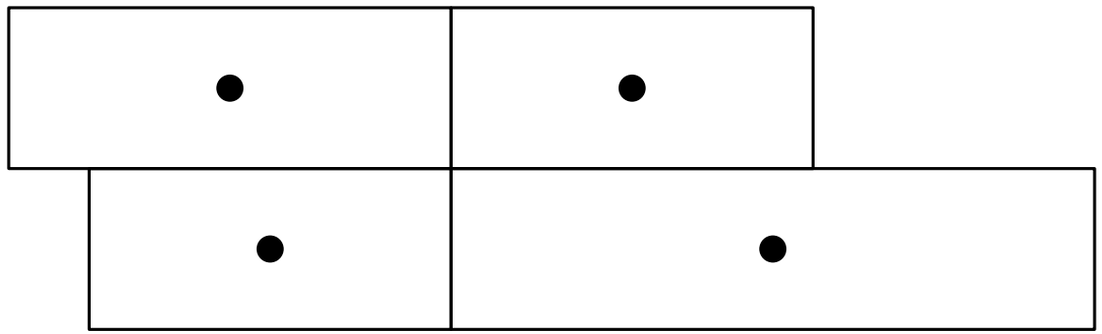

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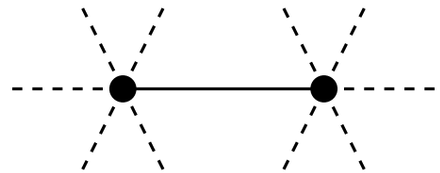

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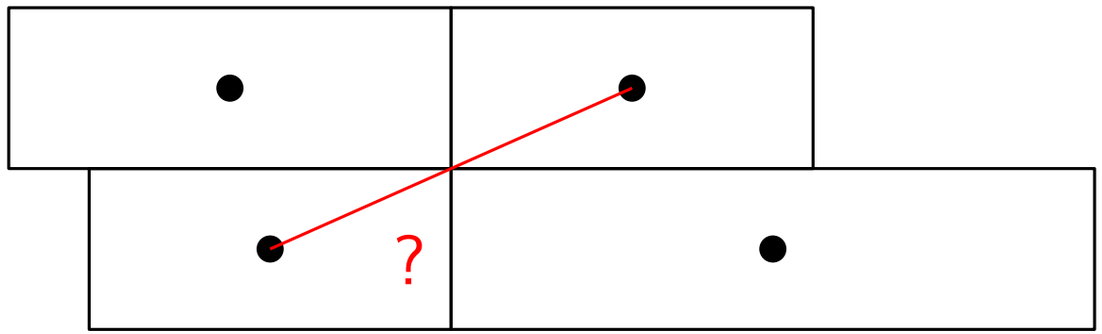

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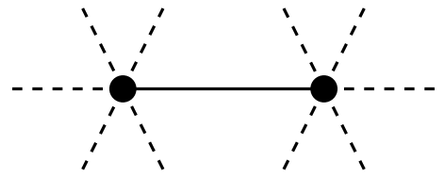

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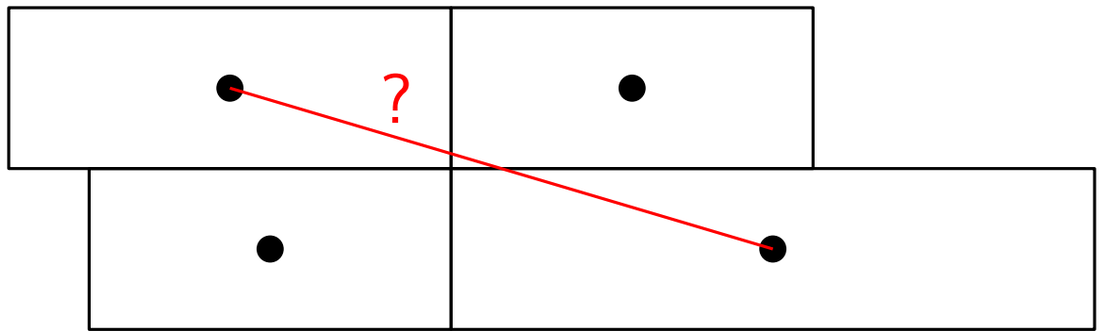

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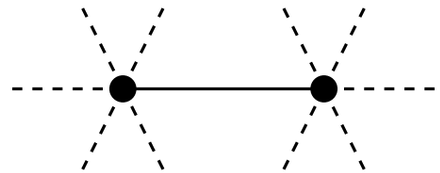

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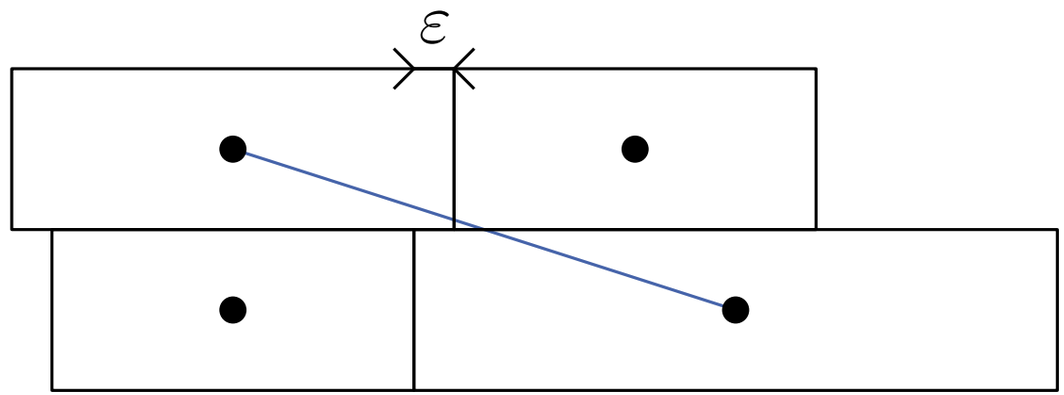

Problem definition

Contact representations of planar graphs are a well-studied topic in graph theory, graph drawing and computational geometry. Vertices are represented by

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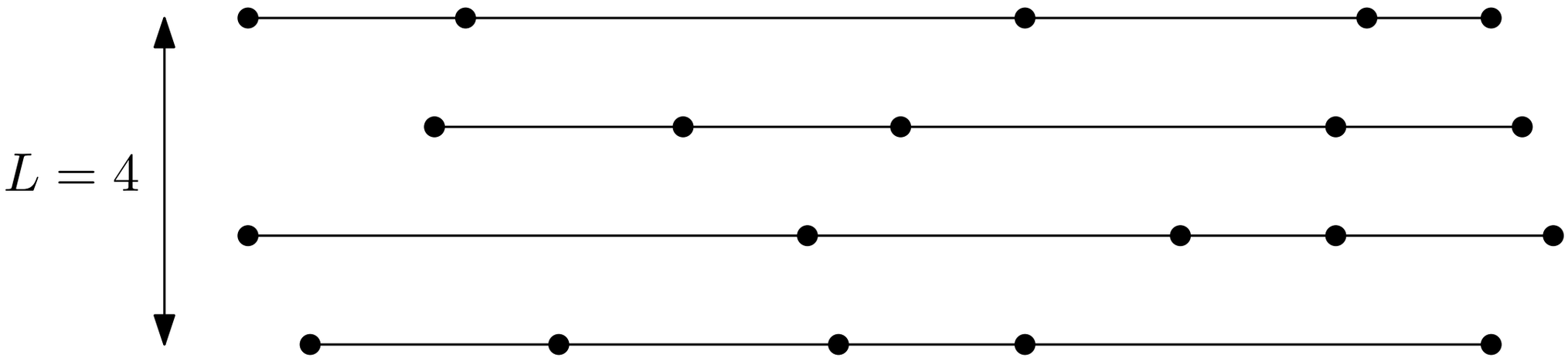

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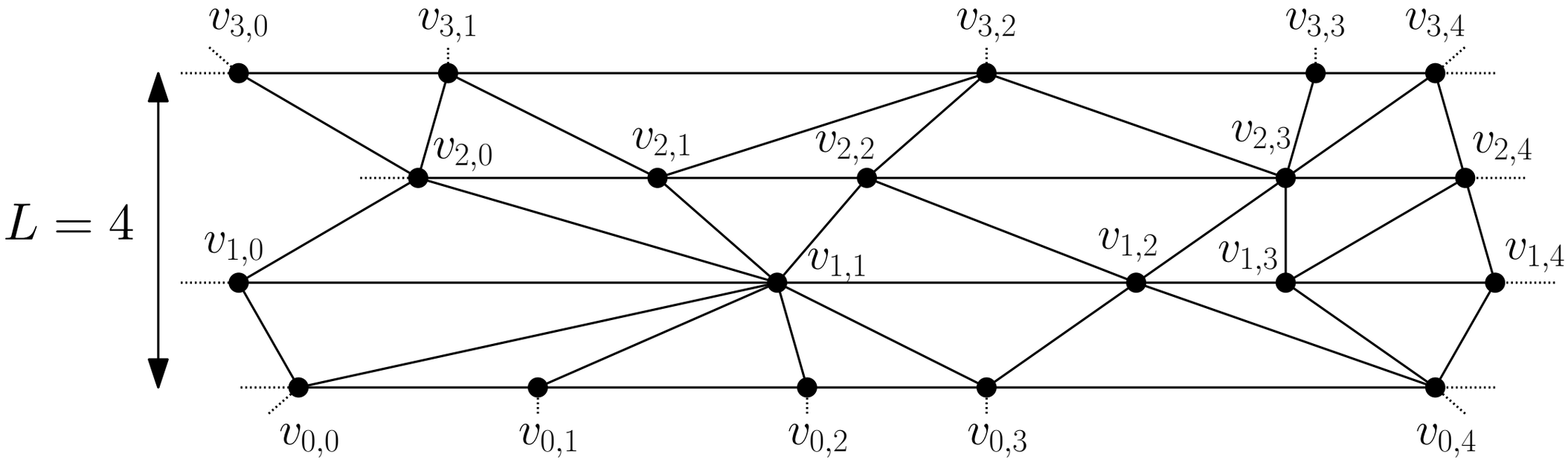
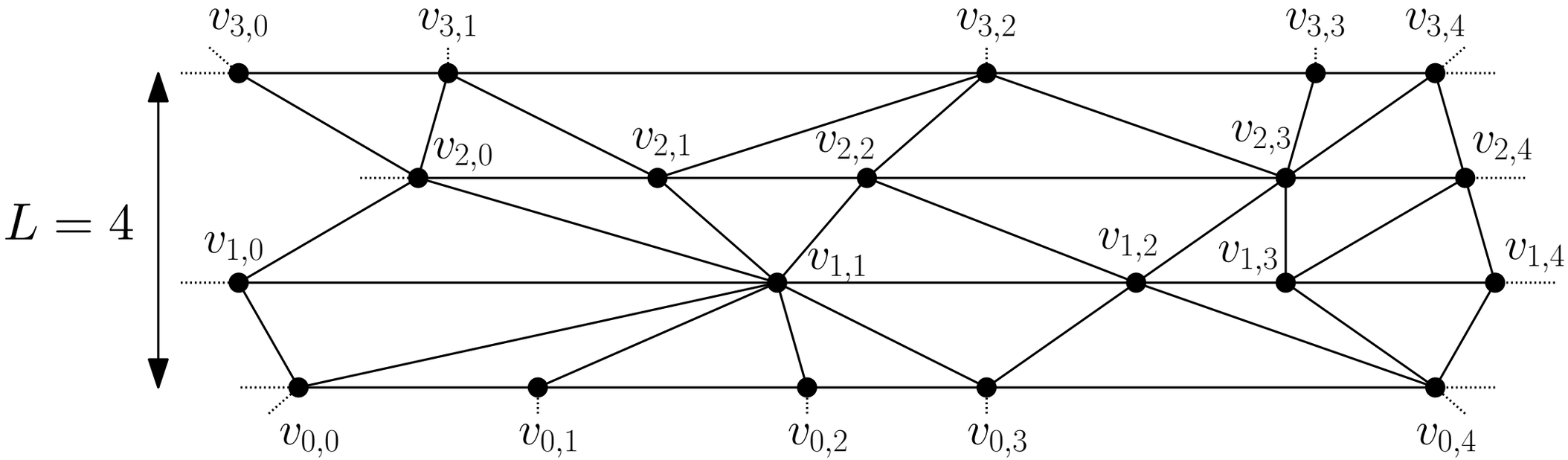


Problem definition

$G = (V, E)$ layered graph with weight function $w : V \to \mathbb{R}^+$

Vertex weight $\iff$ Rectangle width



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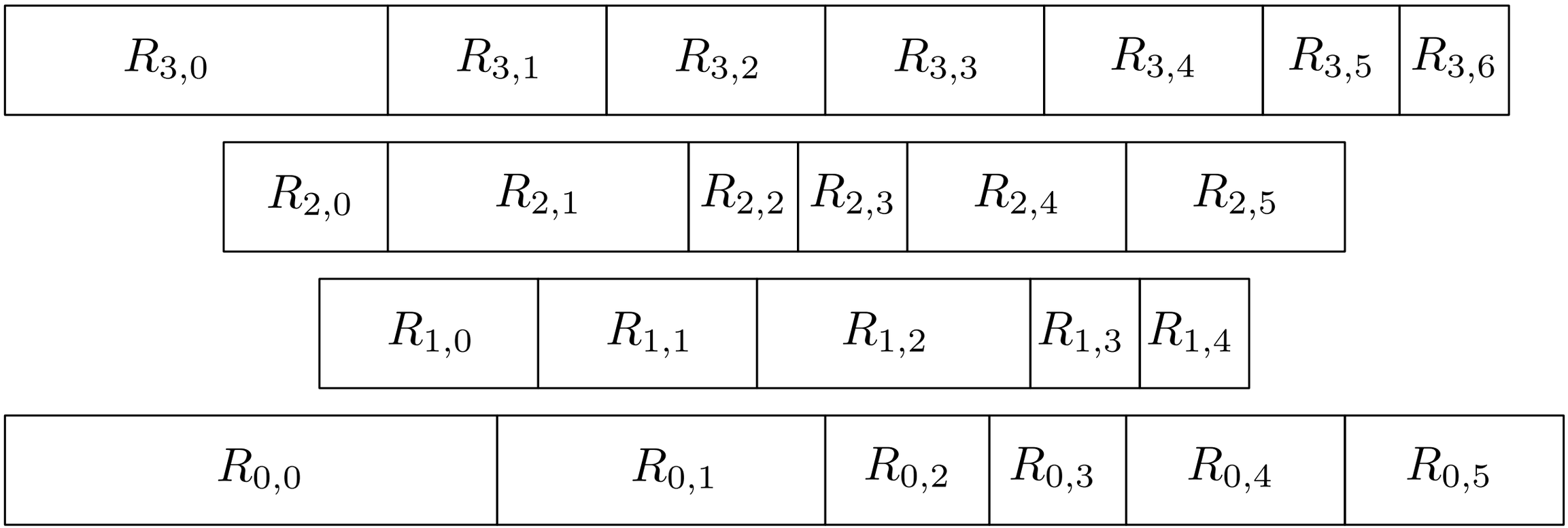

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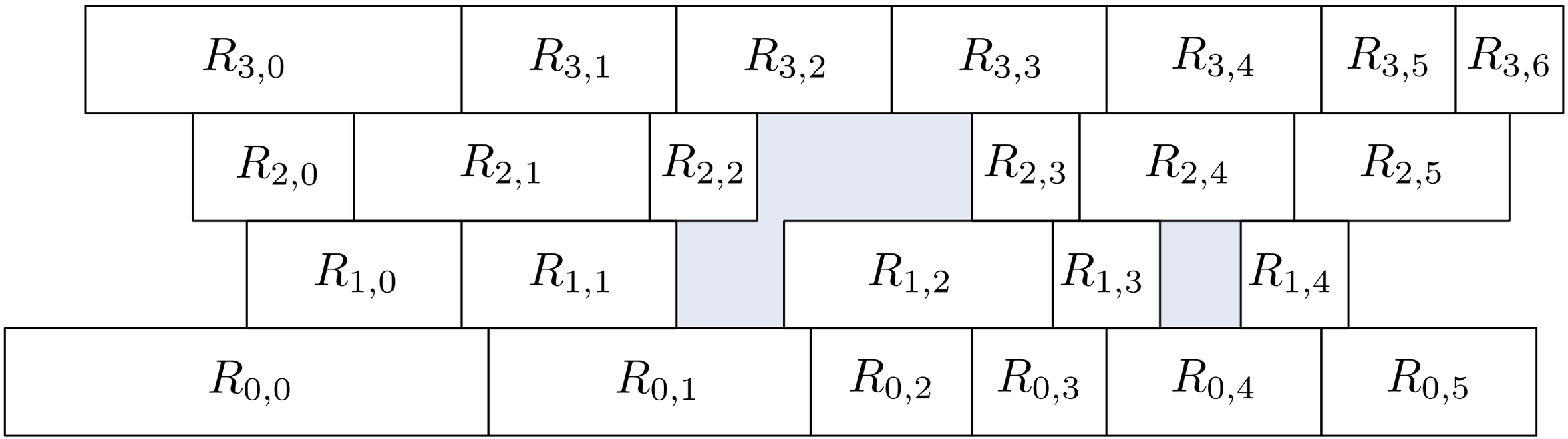
- Contact maximization

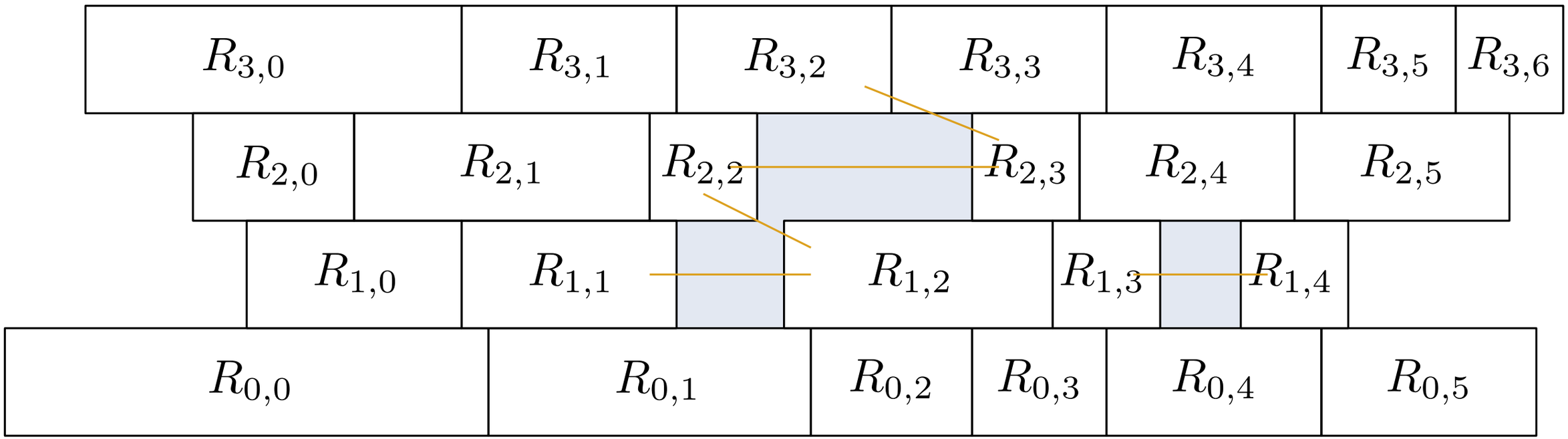


Problem definition

- Contact maximization

Maximize the amount of edges in G realised as box contacts

$\iff$ Maximize p when $\forall e \in E, w(e) = 1$



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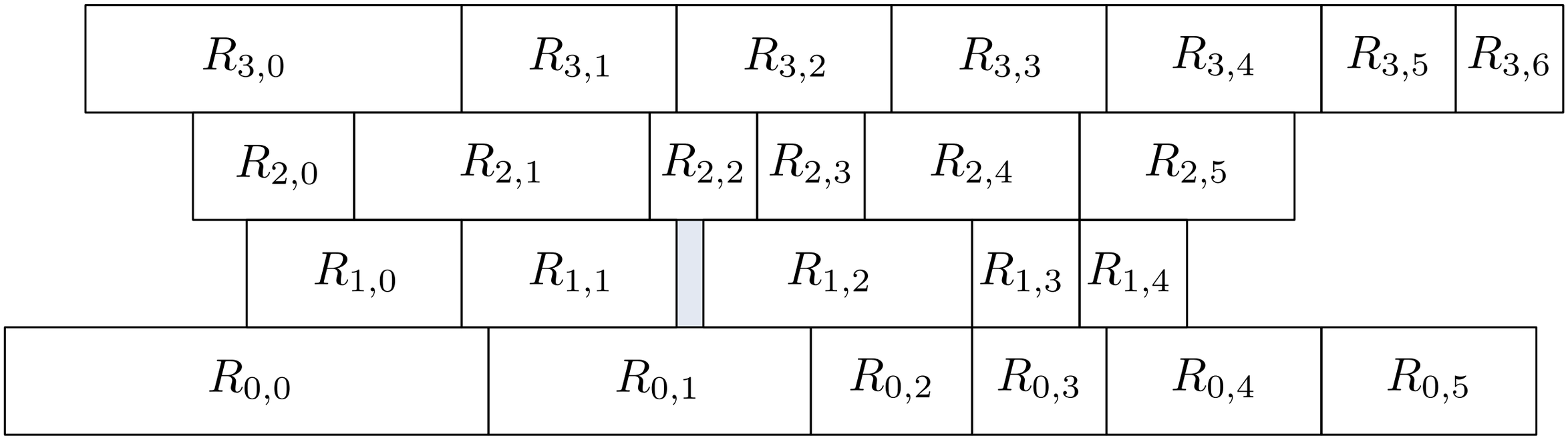

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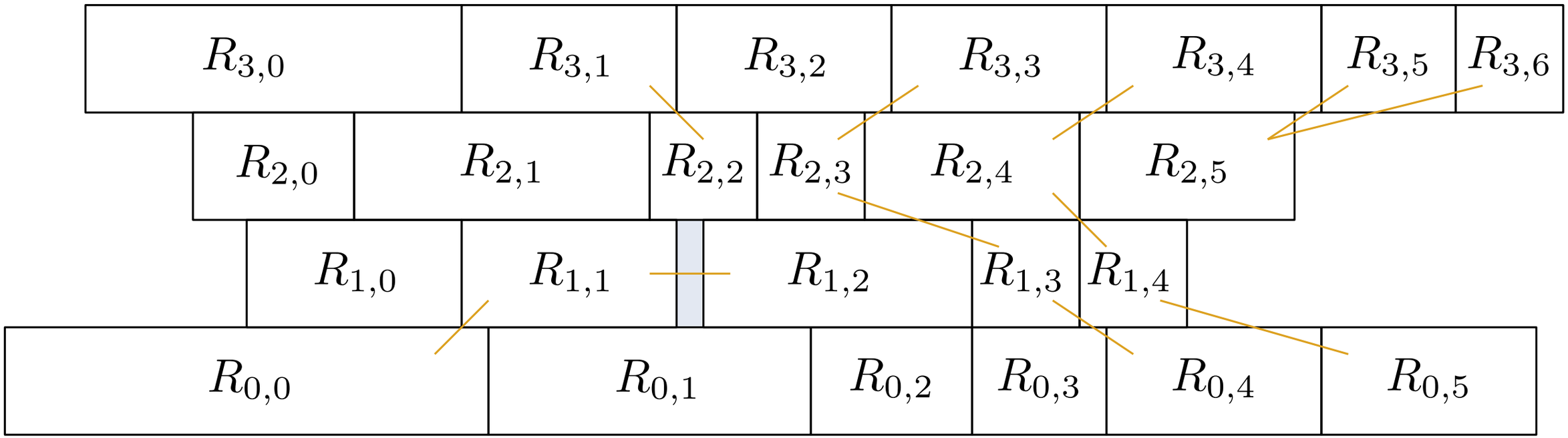
Minimize the total gap width in the representation



Overview

Contact maximization	Area minimization
Greedy algorithm for $L = 2$	Flow network
Extension to $L = 3$?	
ILP formulation for $L \geq 3$	

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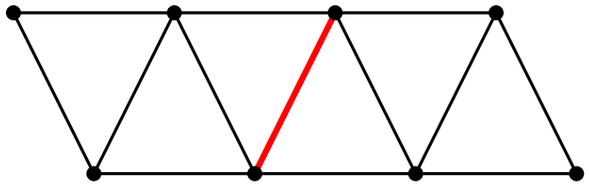
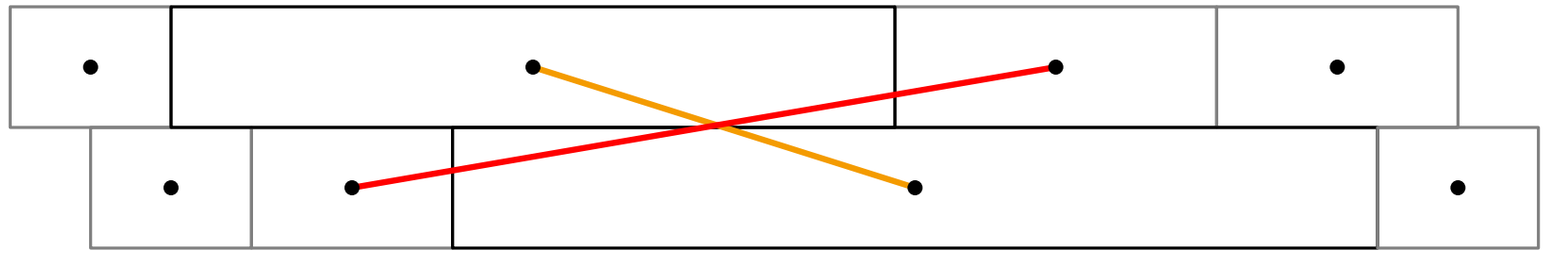
A representation with false contacts can be maximal





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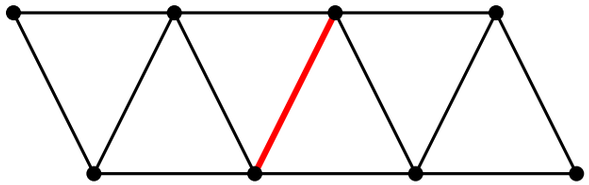
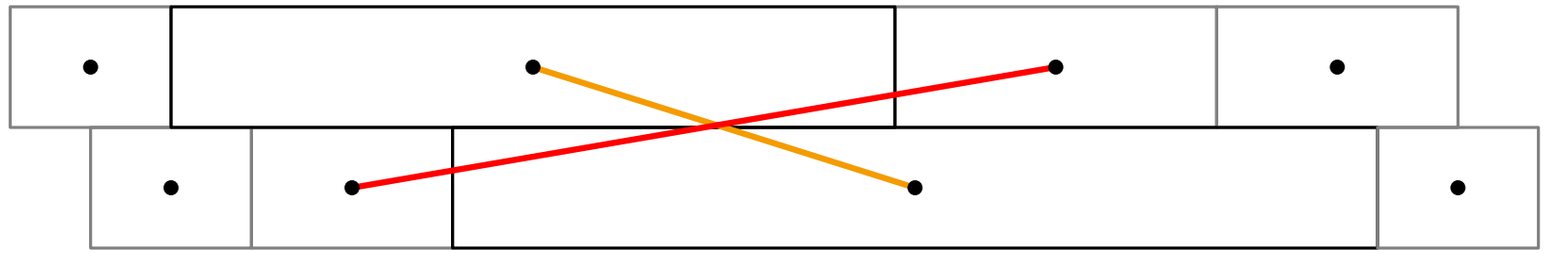
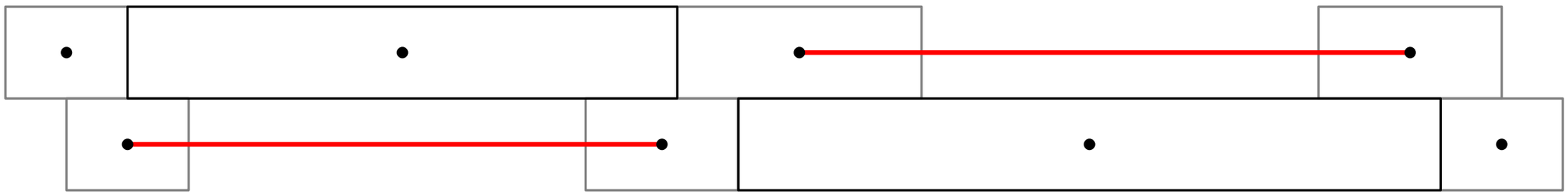
A representation with false contacts can be maximal
$\rightarrow$ not valid





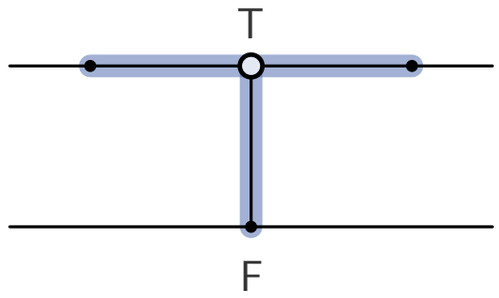
They are forbidden to preserve edge semantics



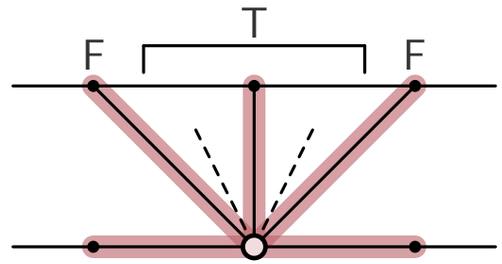
Contact maximization

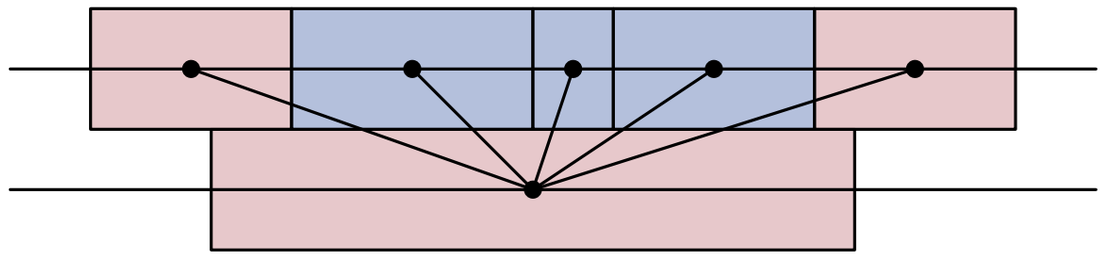
Compute the contact maximal representation **at every step**

T vertex



Fan vertex





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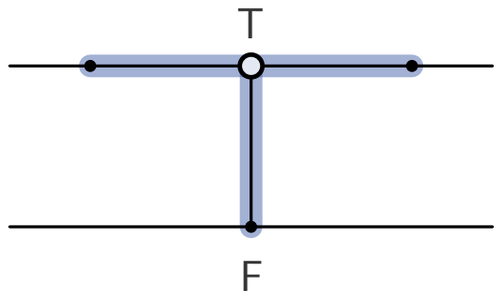

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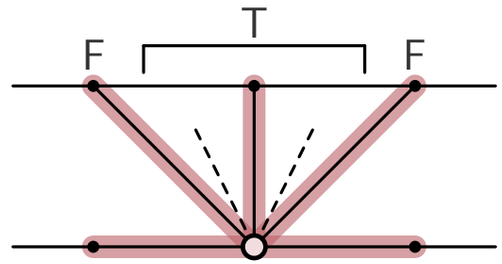

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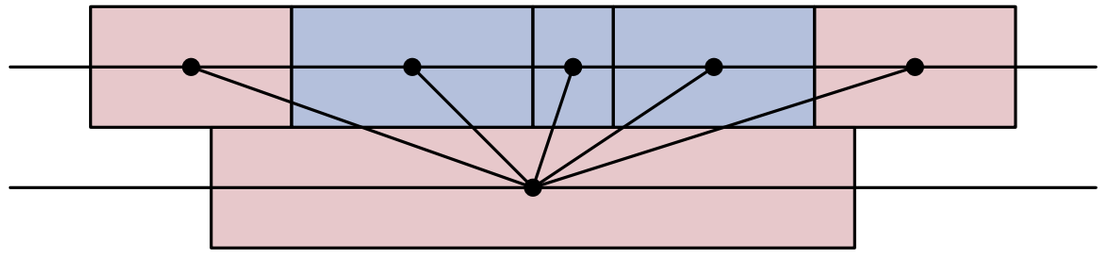

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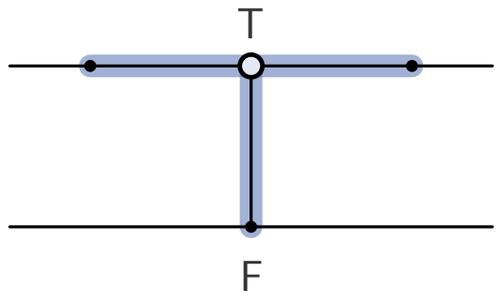

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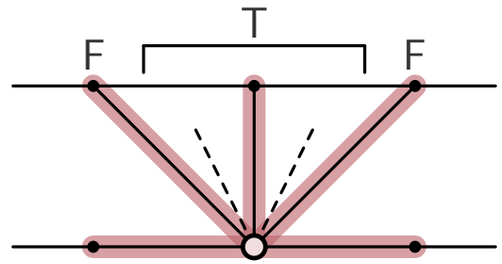

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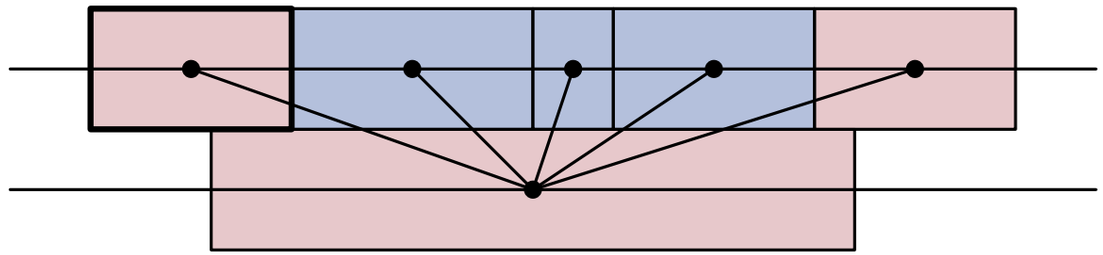

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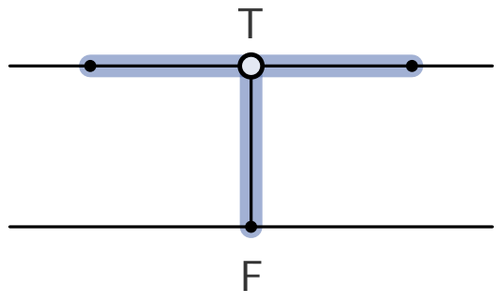

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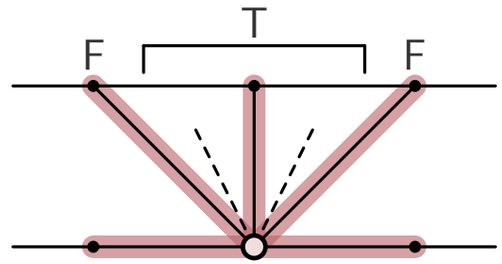

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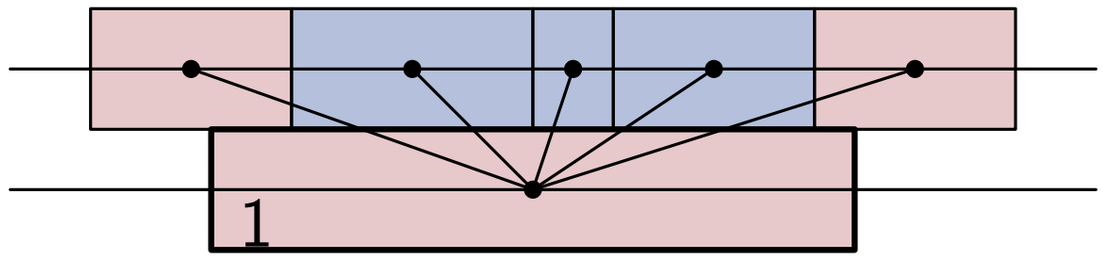

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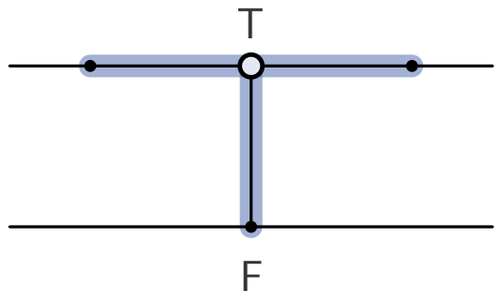

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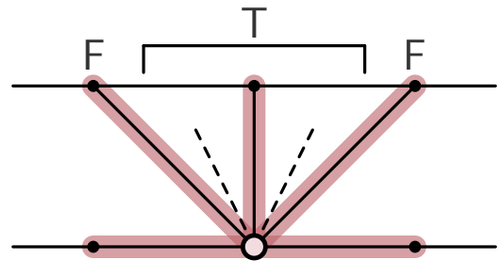

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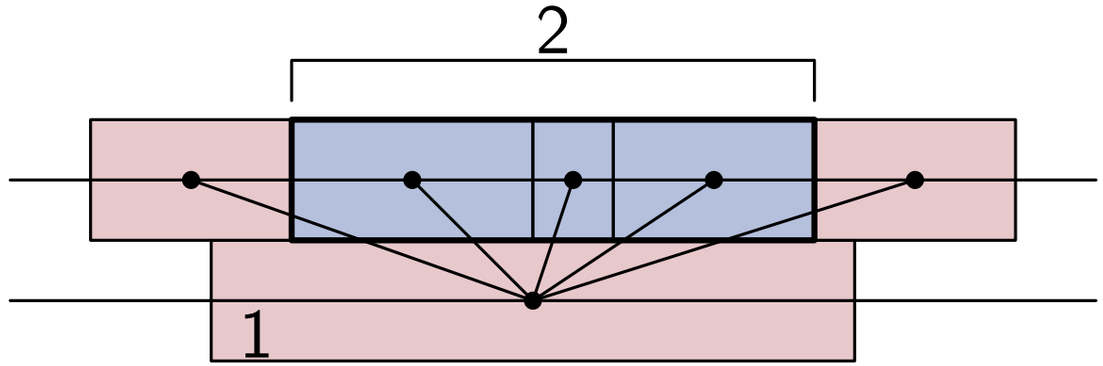

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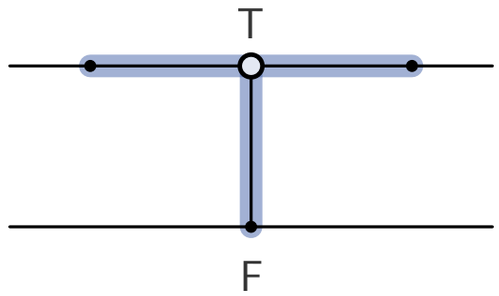
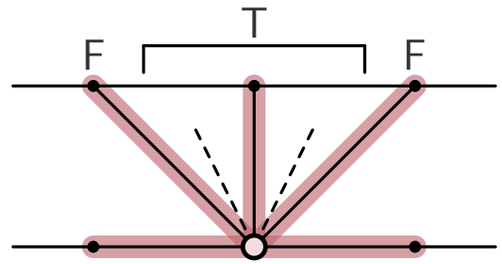
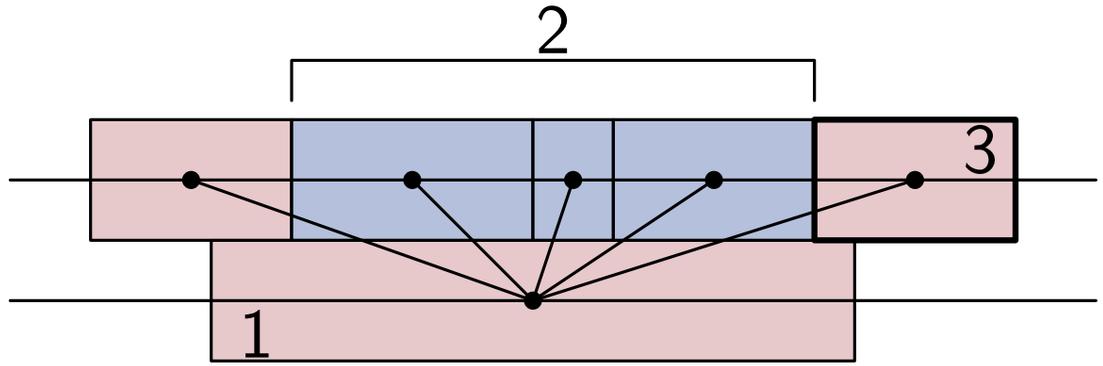


Fan vertex



Algorithm overview

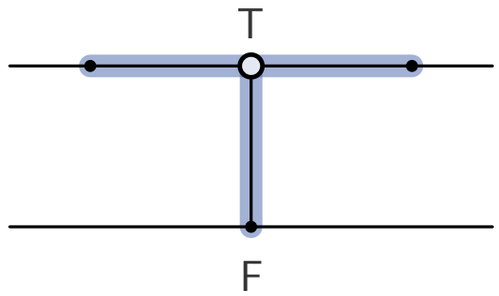
- Layer 1 fan
- Layer 2 T vertices
- Layer 2 fan



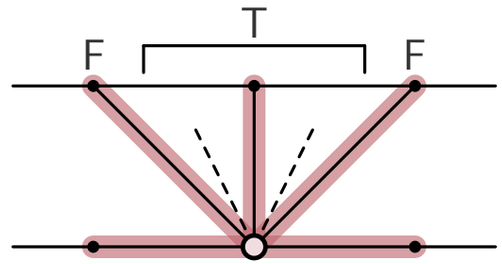
Contact maximization

Compute the contact maximal representation **at every step**

T vertex

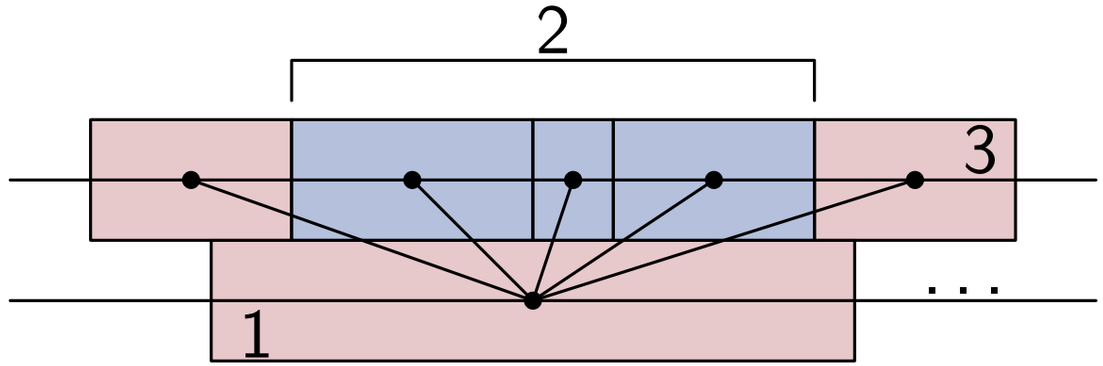


Fan vertex



Algorithm overview

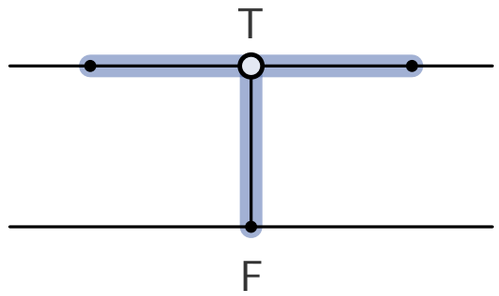
- Layer 1 fan
- Layer 2 T vertices
- Layer 2 fan
- Layer 1 fan/T vertices



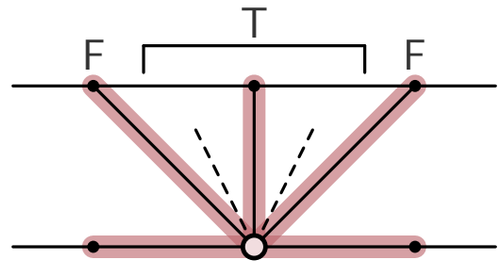
Contact maximization

Compute the contact maximal representation **at every step**

T vertex

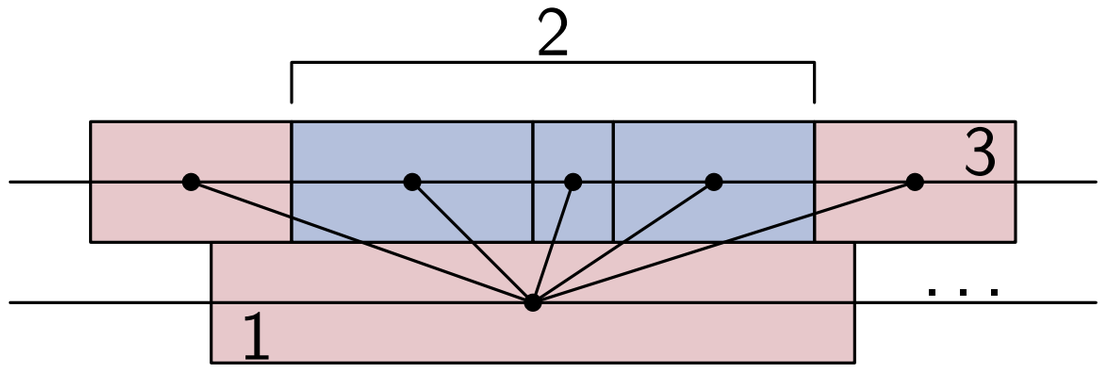


Fan vertex



Algorithm overview

- Layer 1 fan
- Layer 2 T vertices
- Layer 2 fan
- Layer 1 fan/T vertices



$\rightarrow$ Inactive layer ends with a fan

Contact maximization

Compute the contact maximal representation **at every step**

Contact maximization

Compute the contact maximal representation **at every step**

$\rightarrow$ 2 contacts at most per step, 1 vertical, 1 horizontal

Contact maximization

Compute the contact maximal representation **at every step**

$\rightarrow$ 2 contacts at most per step, 1 vertical, 1 horizontal

$\rightarrow$ 3 rules when 2 contacts are not immediate

Contact maximization

Compute the contact maximal representation **at every step**

$\rightarrow$ 2 contacts at most per step, 1 vertical, 1 horizontal

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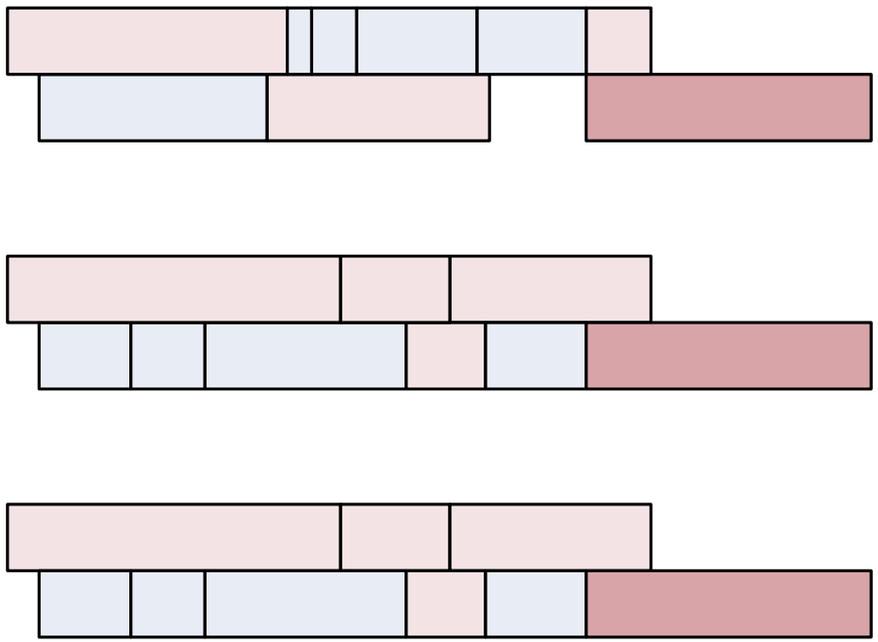

Contact maximization

Compute the contact maximal representation **at every step**

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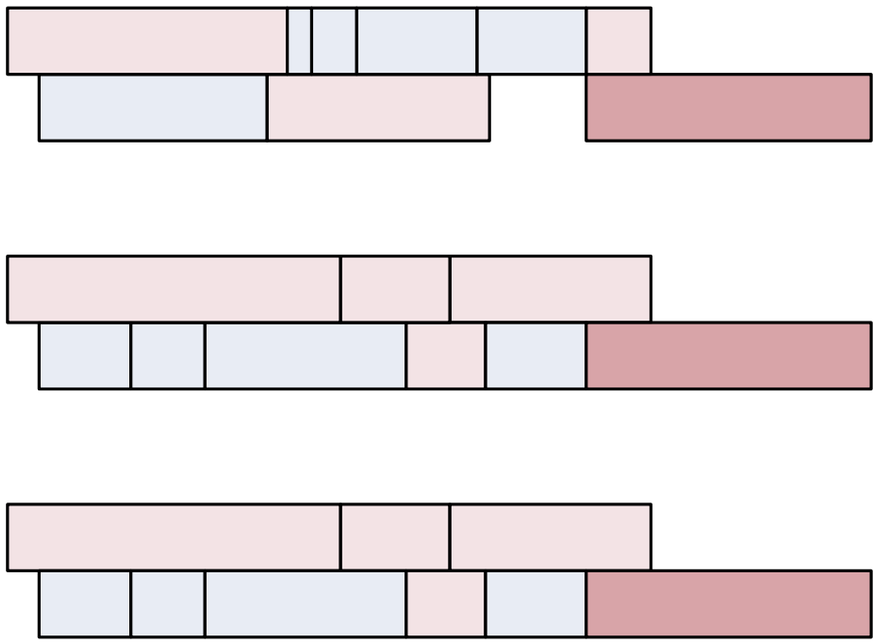

Put the new rectangle as far left as possible

Contact maximization

Compute the contact maximal representation **at every step**

$\rightarrow$ 2 contacts at most per step, 1 vertical, 1 horizontal

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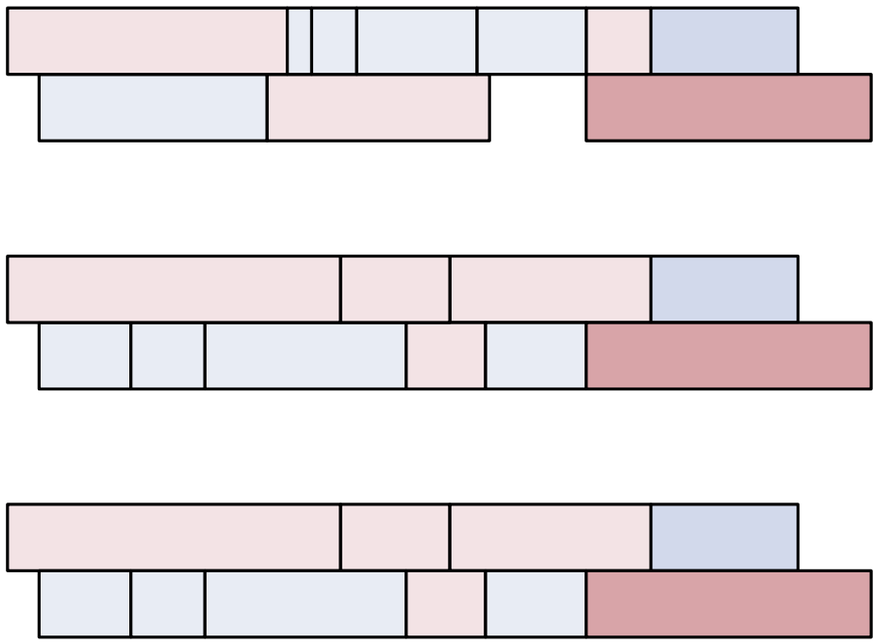

Contact maximization

Compute the contact maximal representation **at every step**

$\rightarrow$ 2 contacts at most per step, 1 vertical, 1 horizontal

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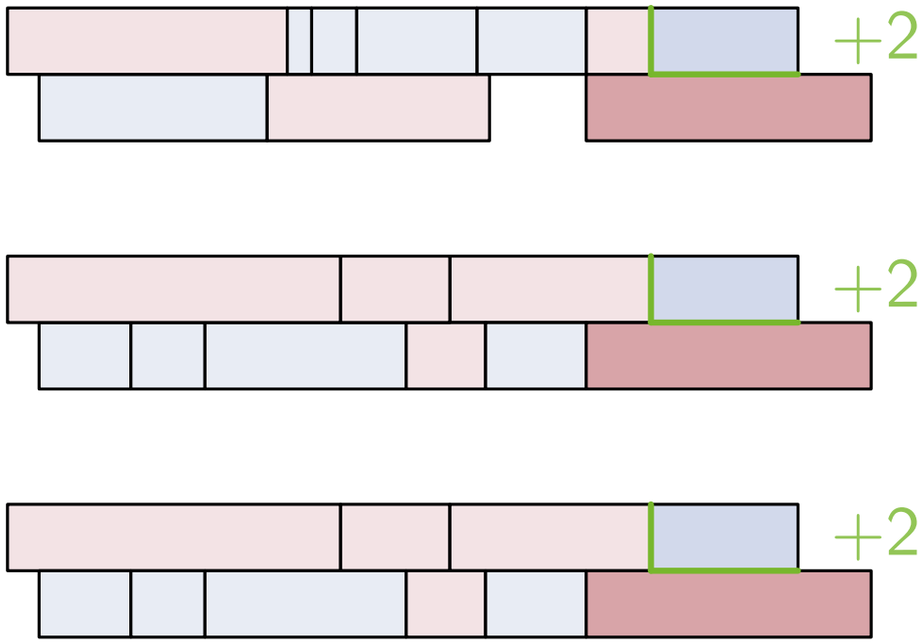

Contact maximization

Compute the contact maximal representation **at every step**

$\rightarrow$ 2 contacts at most per step, 1 vertical, 1 horizontal

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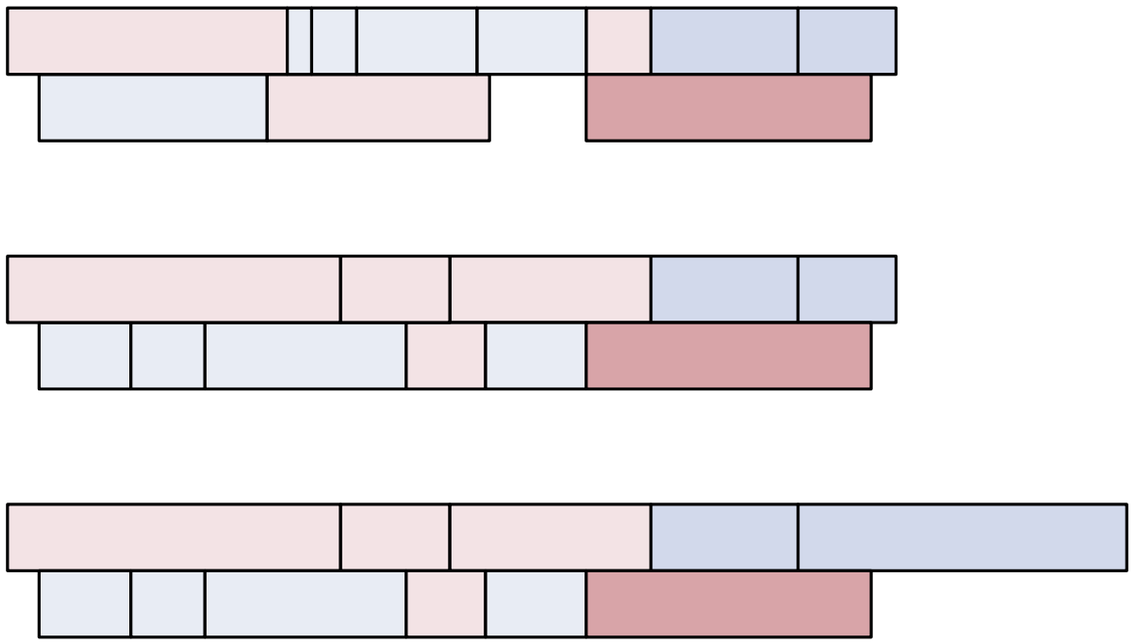
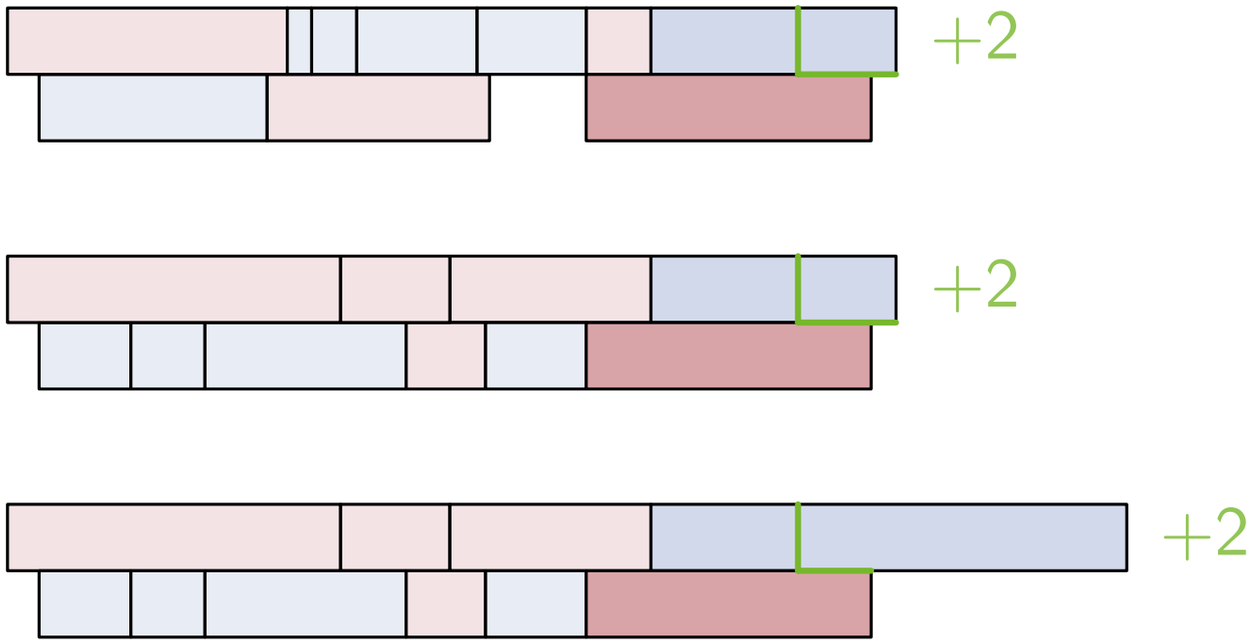


Contact maximization

Compute the contact maximal representation **at every step**

$\rightarrow$ 2 contacts at most per step, 1 vertical, 1 horizontal

$\rightarrow$ 3 rules when 2 contacts are not immediate

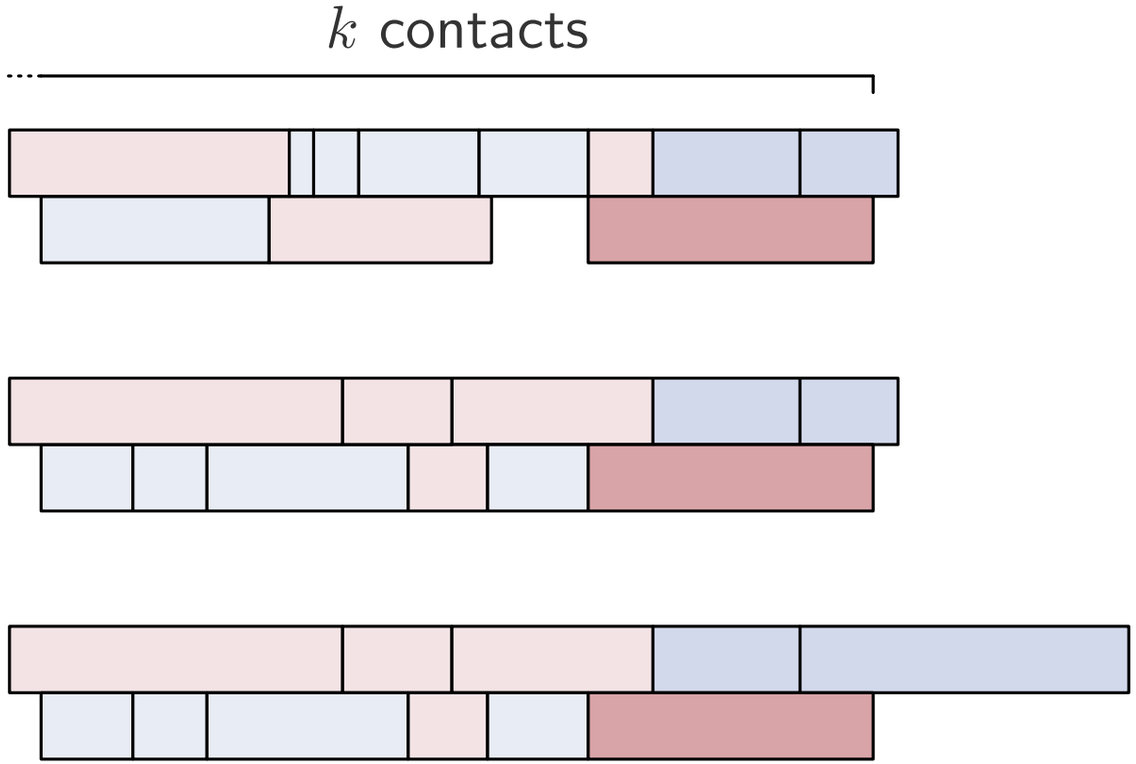


Contact maximization

Compute the contact maximal representation **at every step**

$\rightarrow$ 2 contacts at most per step, 1 vertical, 1 horizontal

$\rightarrow$ 3 rules when 2 contacts are not immediate



Contact maximization

Compute the contact maximal representation **at every step**

$\rightarrow$ 2 contacts at most per step, 1 vertical, 1 horizontal

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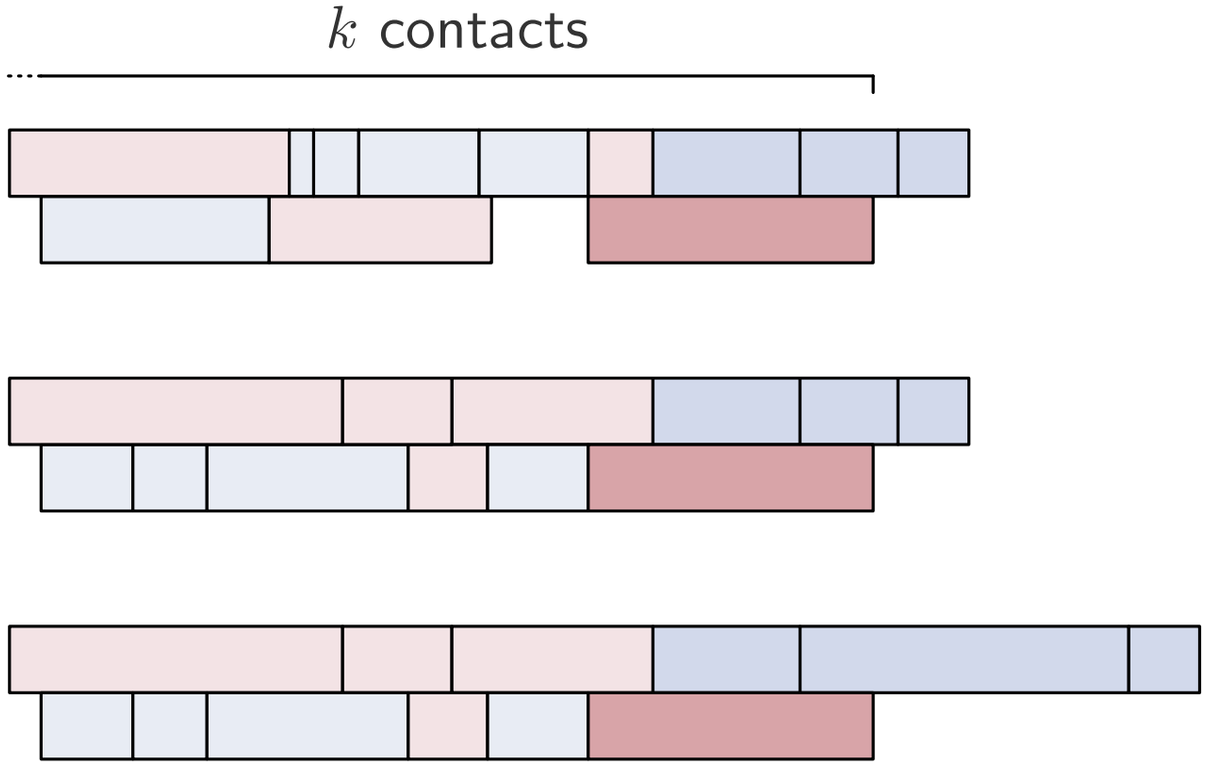

Contact maximization

Compute the contact maximal representation **at every step**

$\rightarrow$ 2 contacts at most per step, 1 vertical, 1 horizontal

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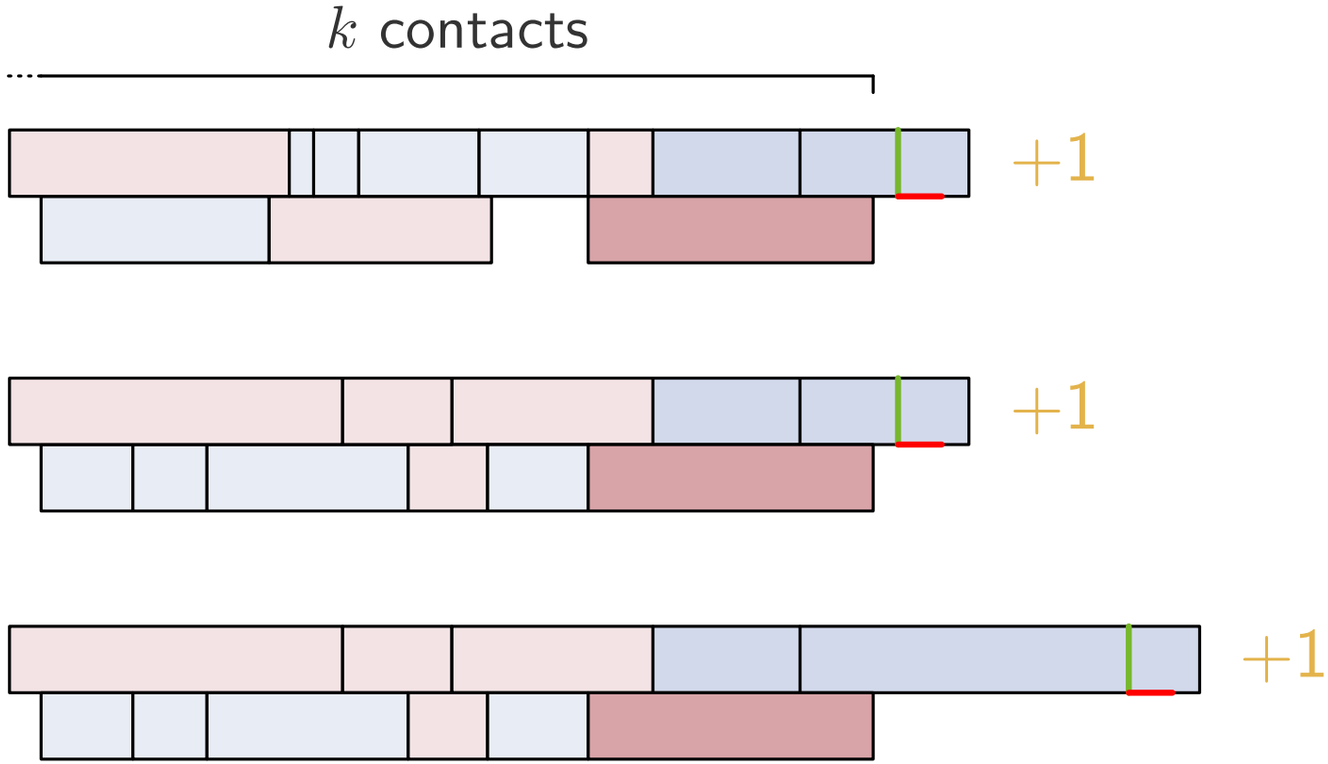

Contact maximization

Compute the contact maximal representation **at every step**

$\rightarrow$ 2 contacts at most per step, 1 vertical, 1 horizontal

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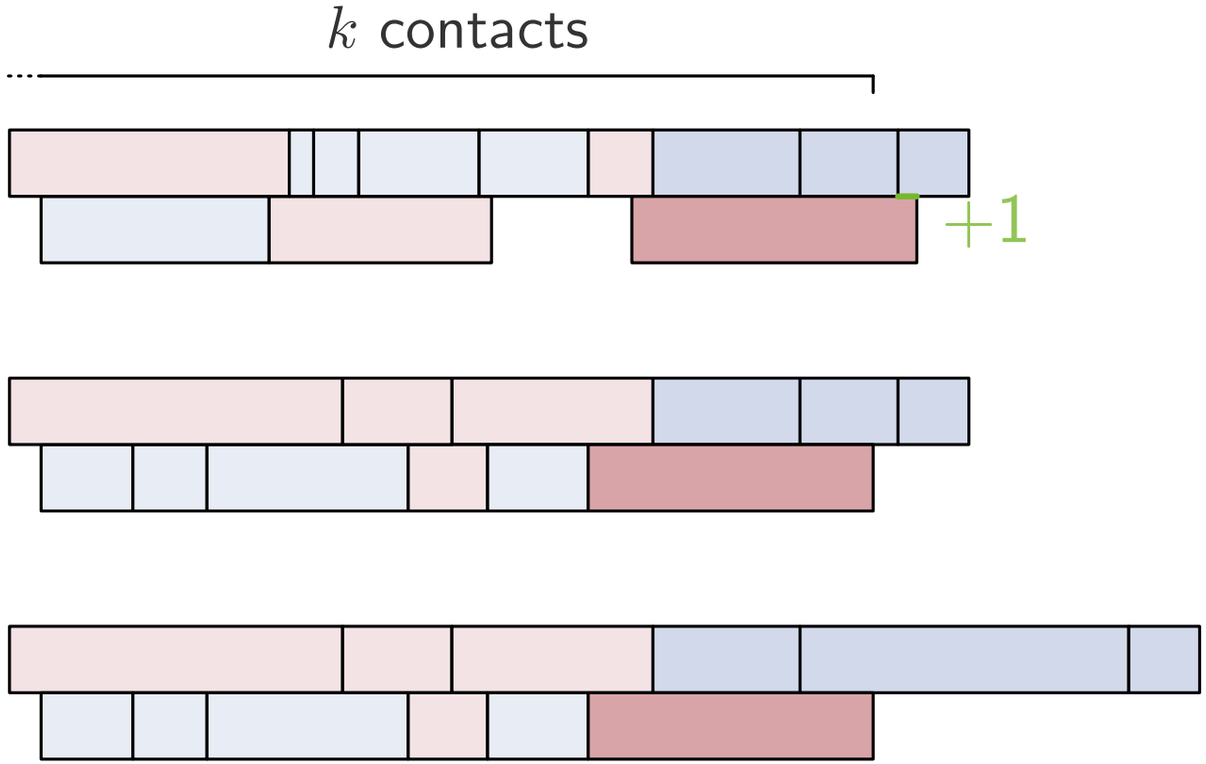

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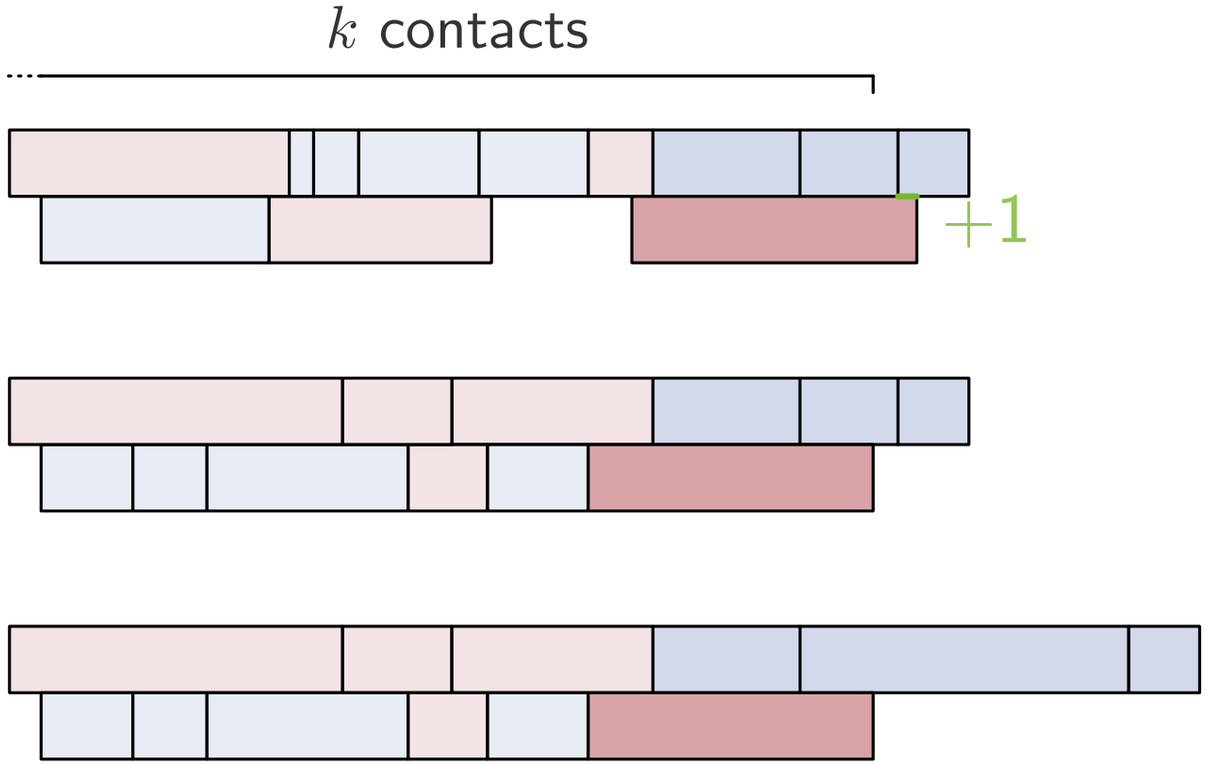

Shift is strictly better
new position is kept
$\rightarrow k + 2$ contacts

Contact maximization

Compute the contact maximal representation **at every step**

$\rightarrow$ 2 contacts at most per step, 1 vertical, 1 horizontal

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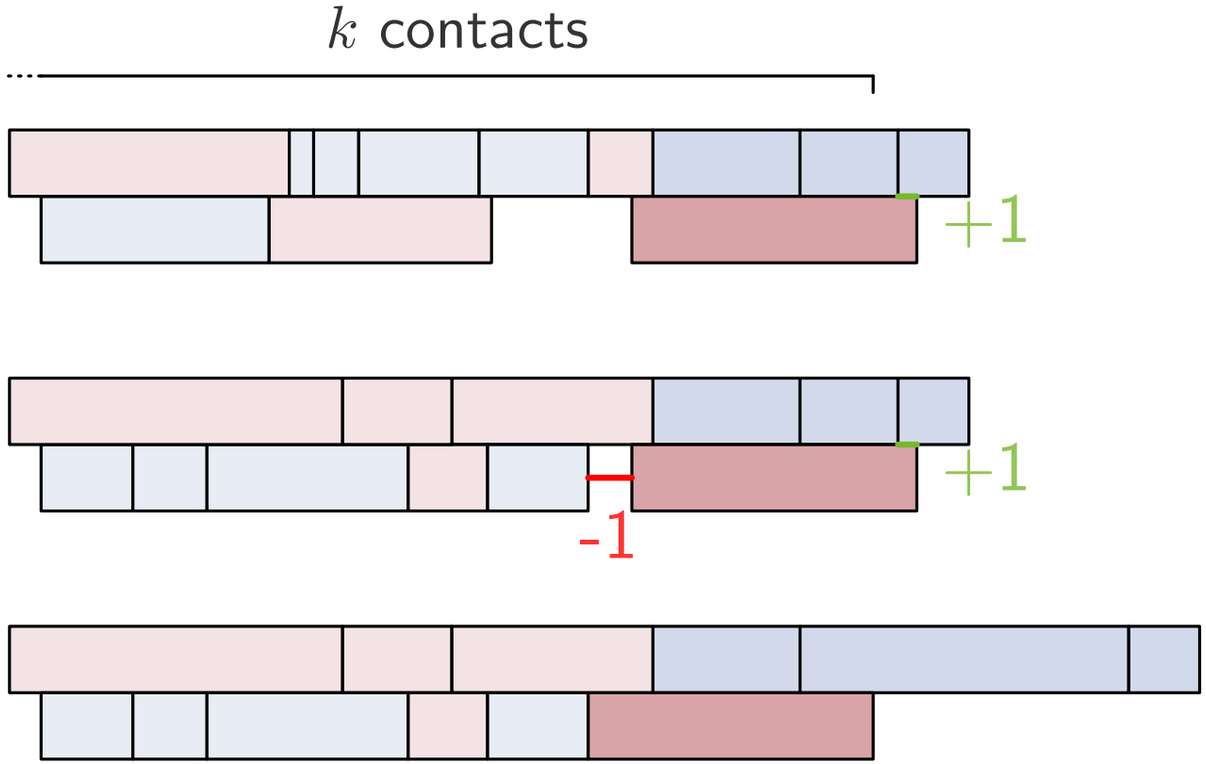

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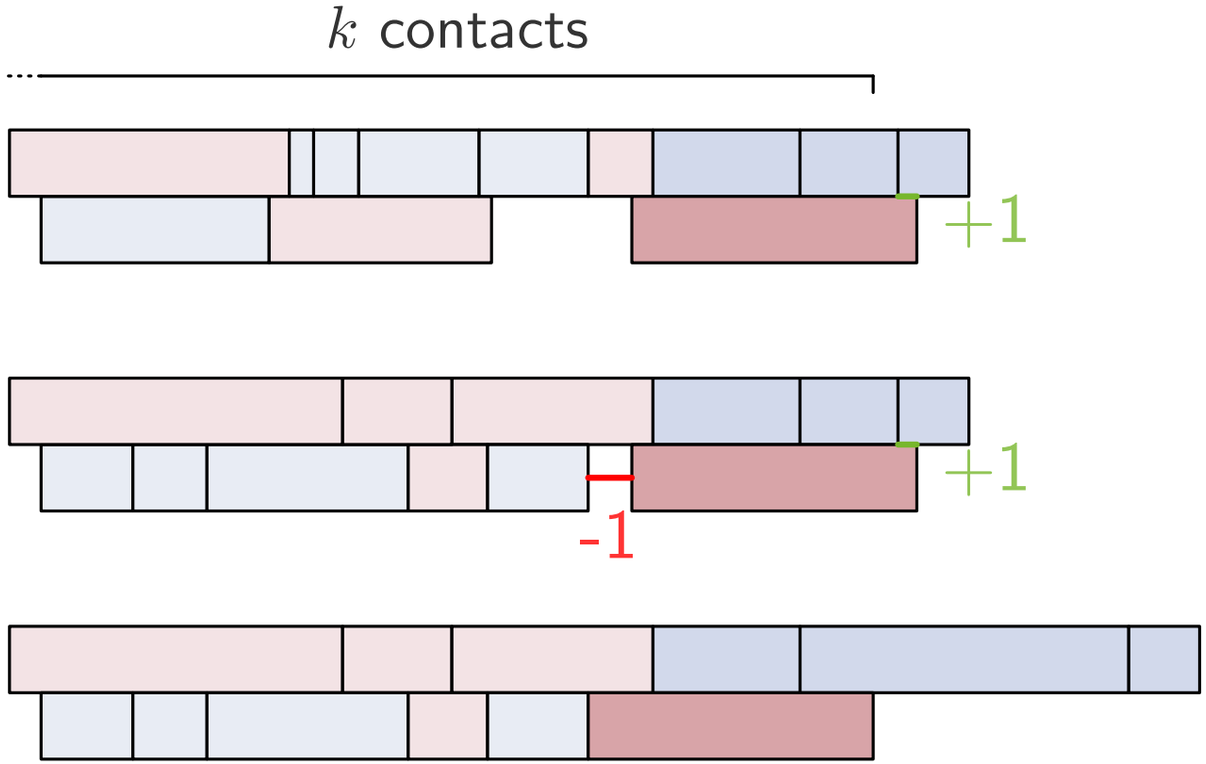

Shift is strictly better
new position is kept
$\rightarrow k + 2$ contacts

Shift is a tie
new position is kept
$\rightarrow k + 1$ contacts

Contact maximization

Compute the contact maximal representation **at every step**

$\rightarrow$ 2 contacts at most per step, 1 vertical, 1 horizontal

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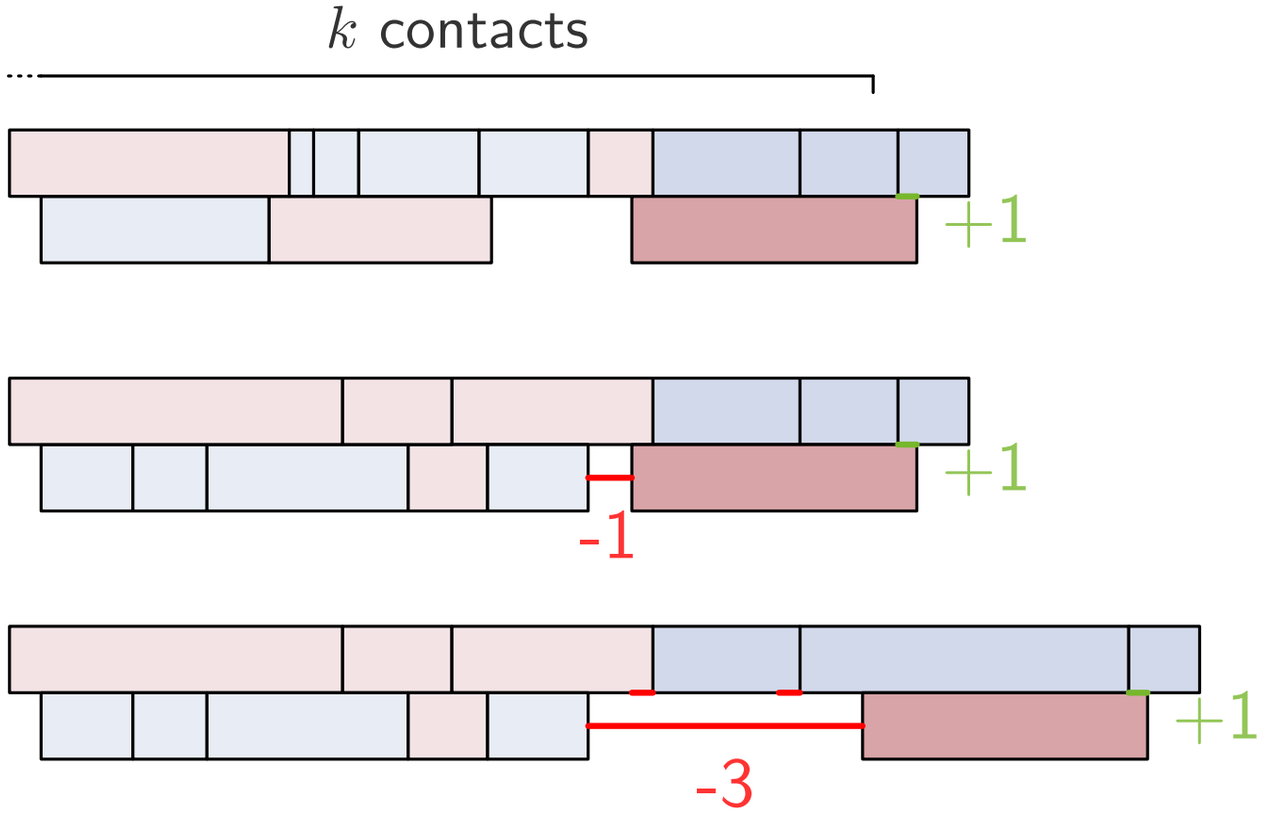

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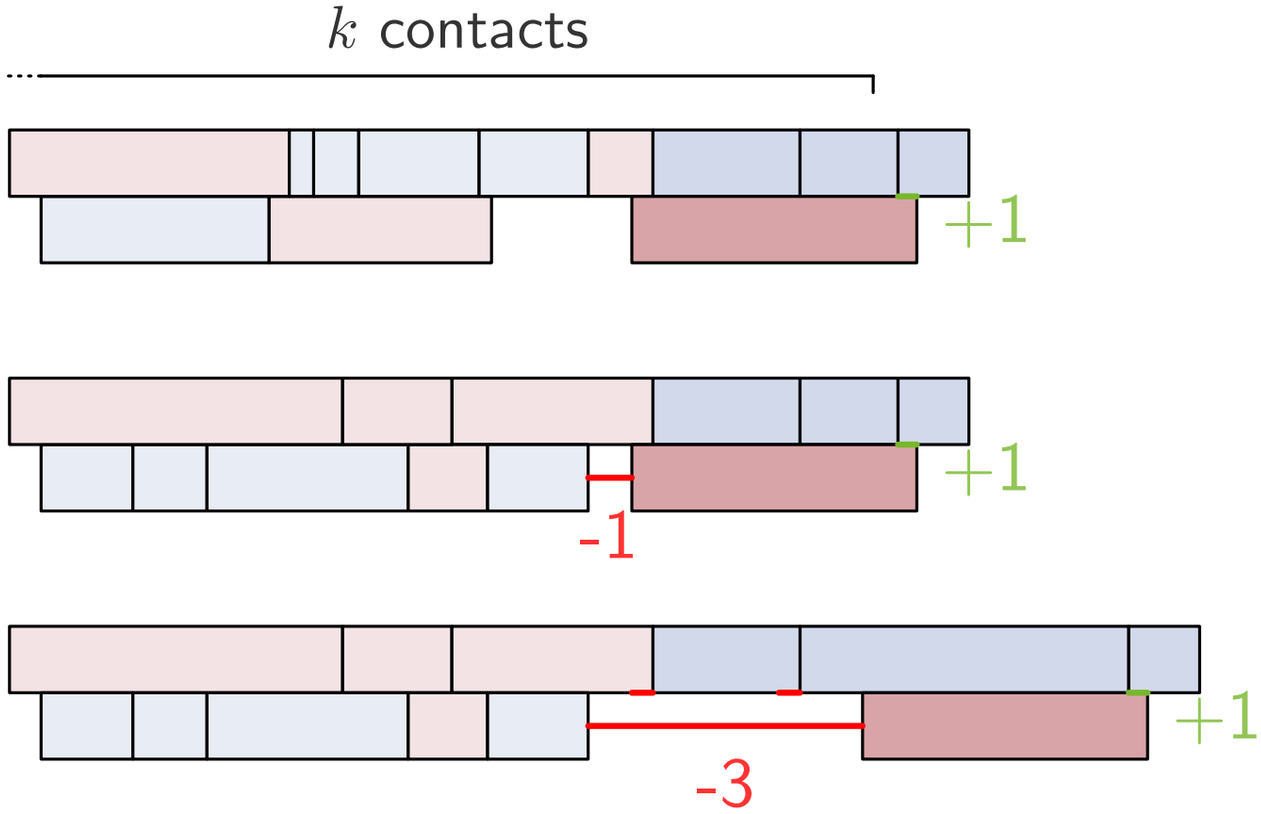

Shift is strictly better
new position is kept
$\rightarrow k + 2$ contacts

Shift is a tie
new position is kept
$\rightarrow k + 1$ contacts

Shift is strictly worse
position is reset
$\rightarrow k + 1$ contacts

Contact maximization

Compute the contact maximal representation **at every step**

$\rightarrow$ 2 contacts at most per step, 1 vertical, 1 horizontal

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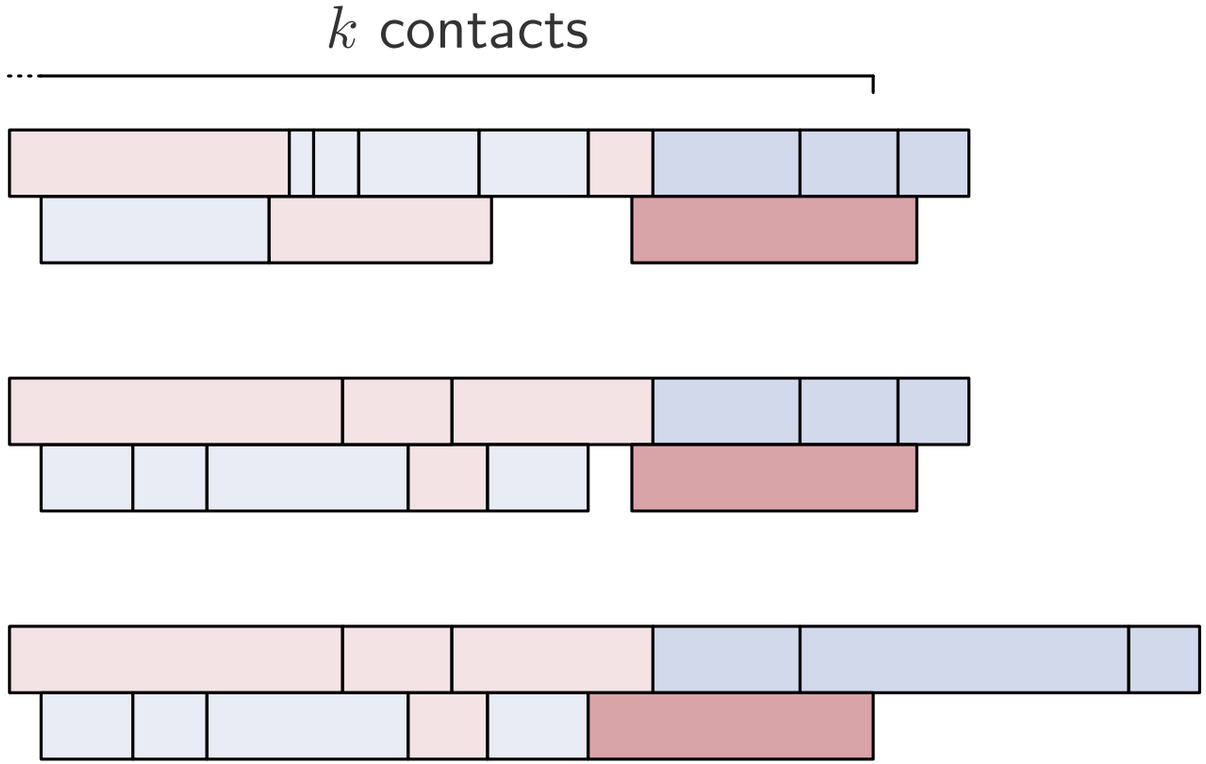

Shift is strictly better
new position is kept
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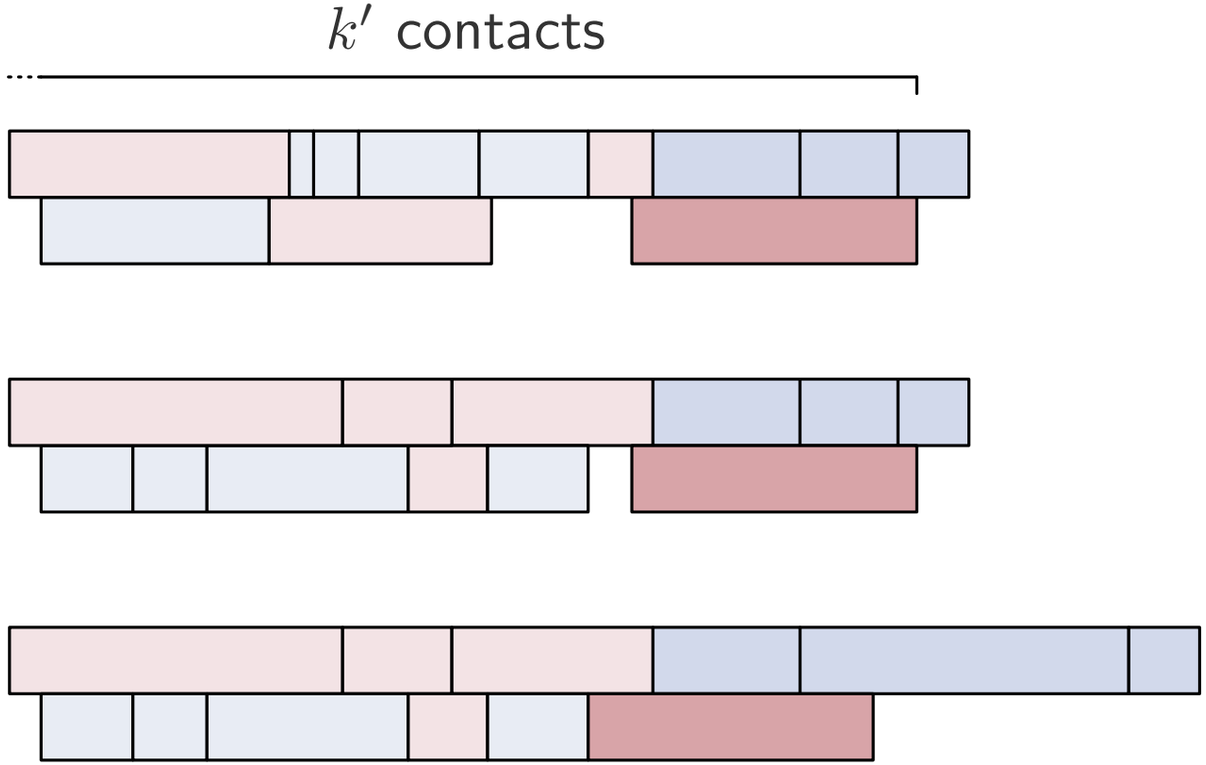

Shift is strictly better
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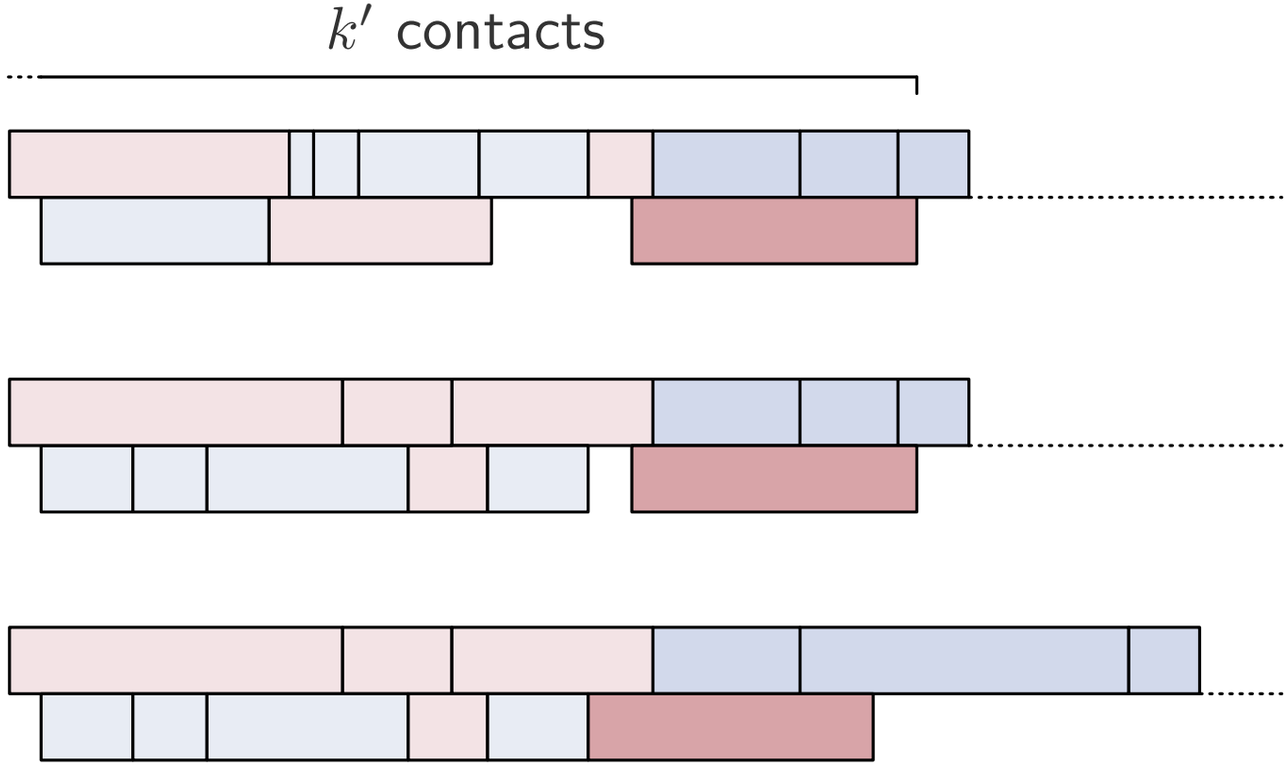

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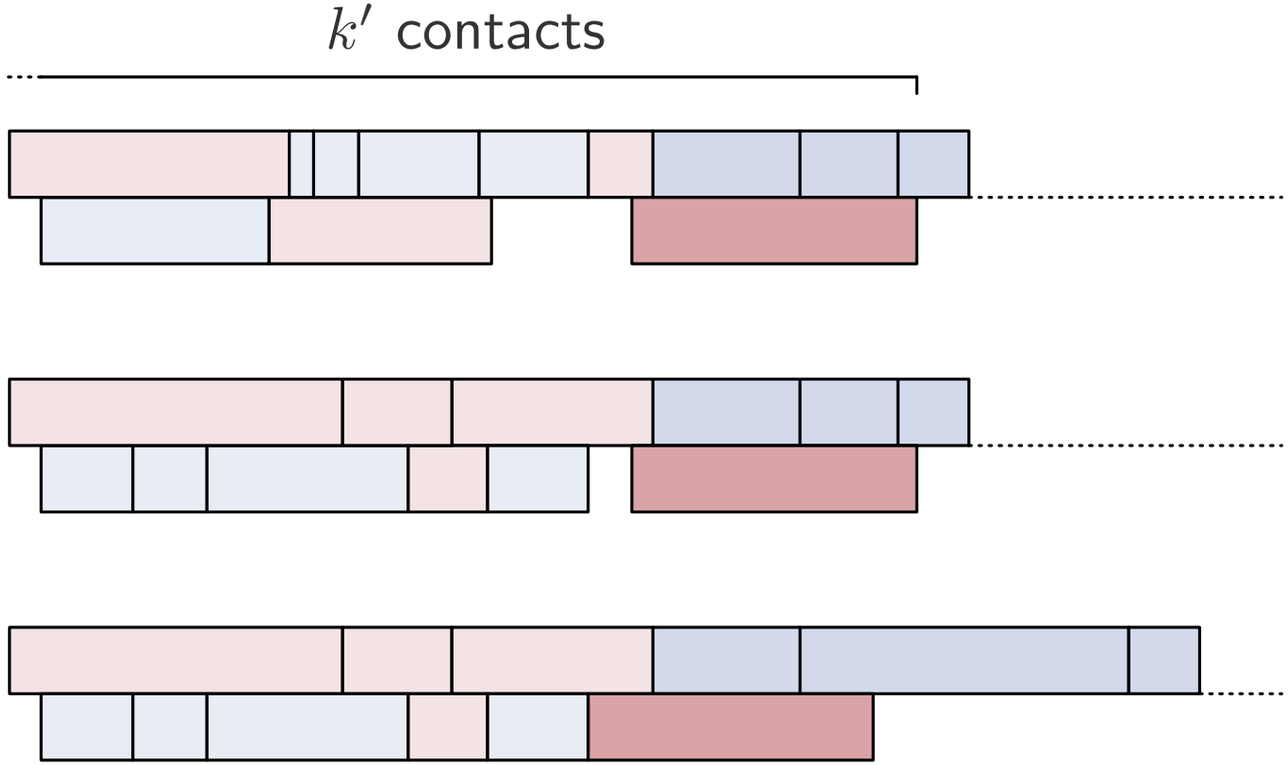

Shift is strictly better
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$\rightarrow k+2$ contacts

Shift is a tie
new position is kept
$\rightarrow k+1$ contacts

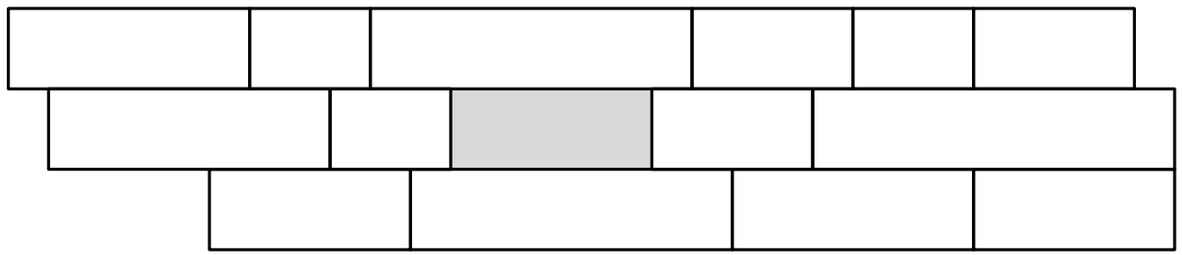
Shift is strictly worse
position is reset
$\rightarrow k+1$ contacts

Theorem 1 :

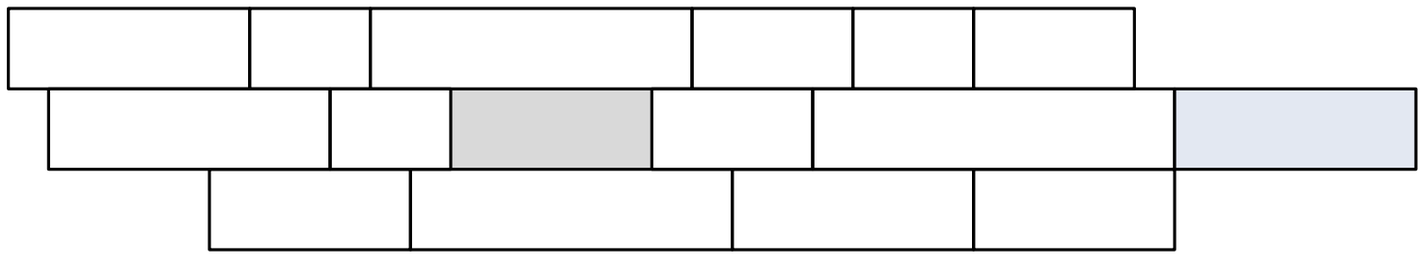
We can maximize the number of realized contacts in $O(n)$ time for $L = 2$.

Extension to $L = 3$

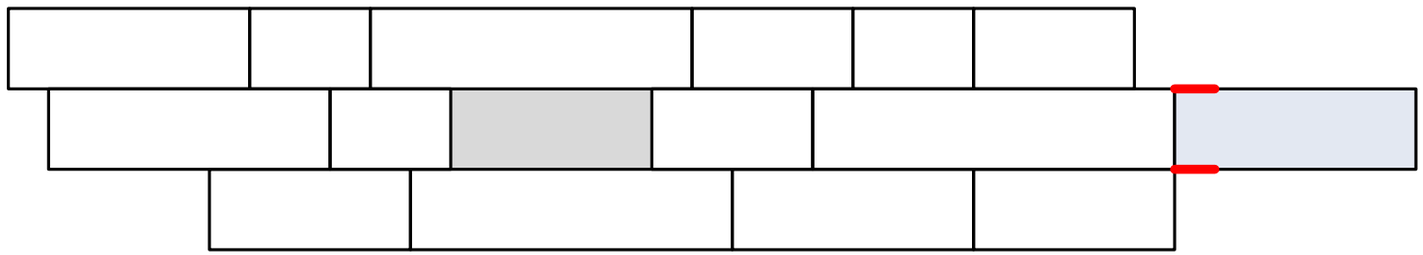
Extension to $L = 3$



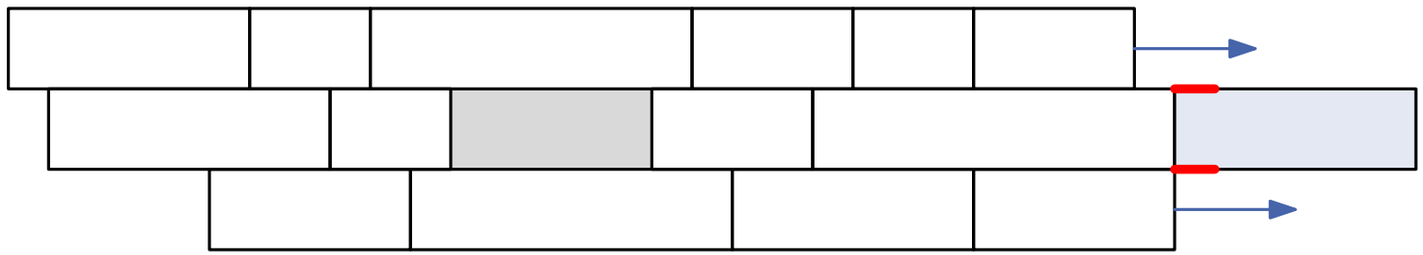
Extension to $L = 3$



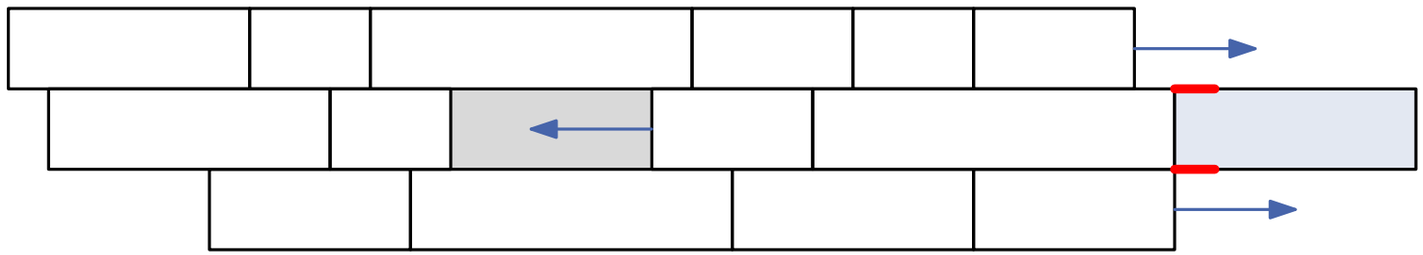
Extension to $L = 3$



Extension to $L = 3$



Extension to $L = 3$



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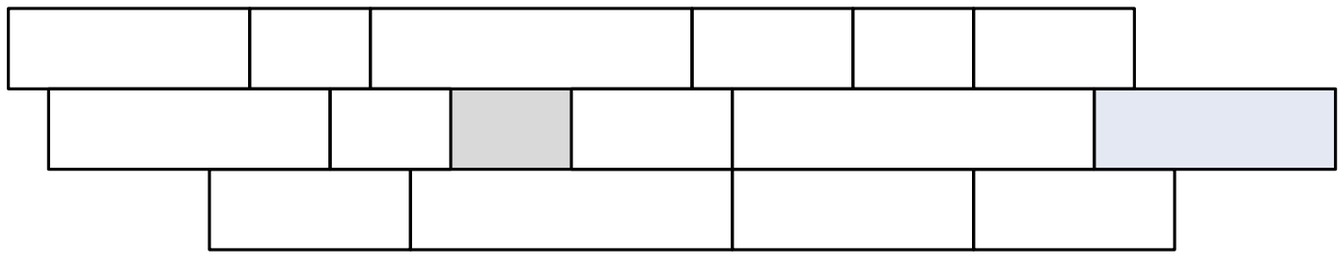
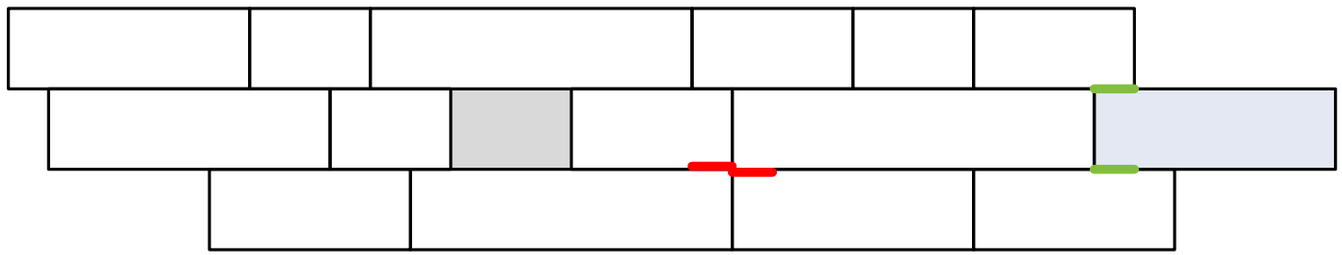
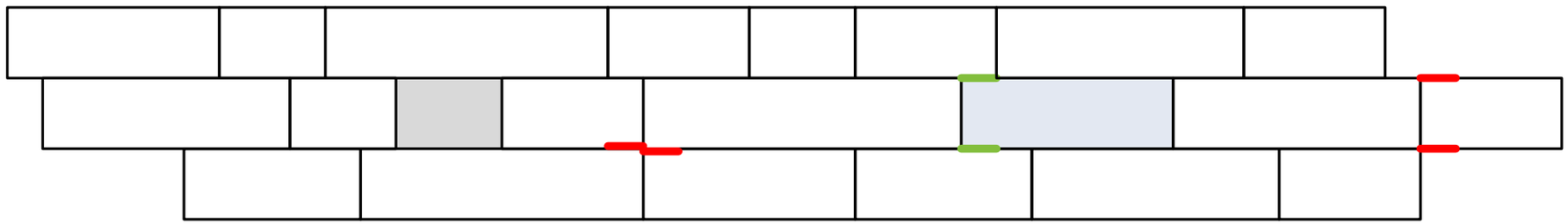


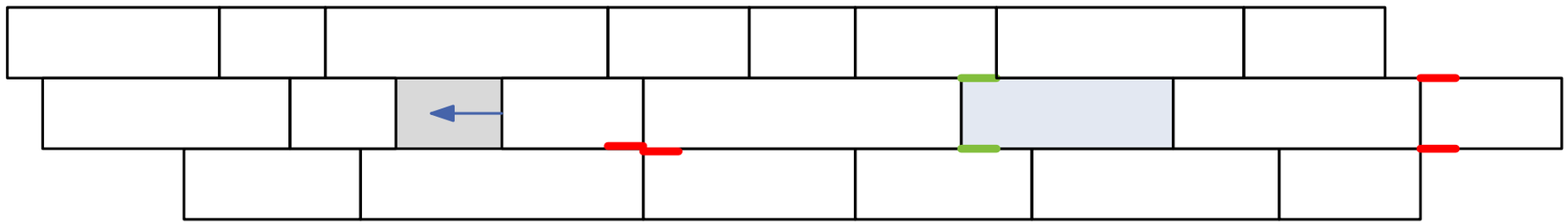
Extension to $L = 3$



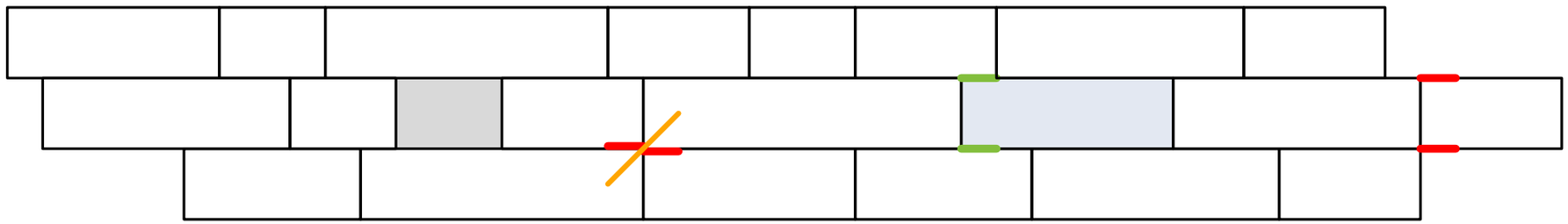
Extension to $L = 3$



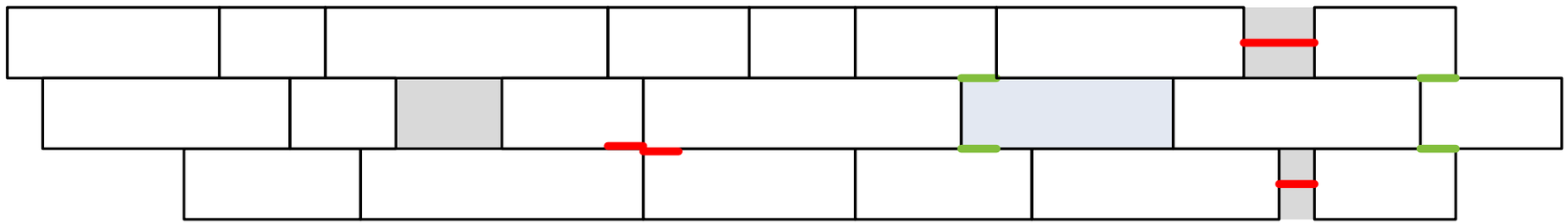
Extension to $L = 3$



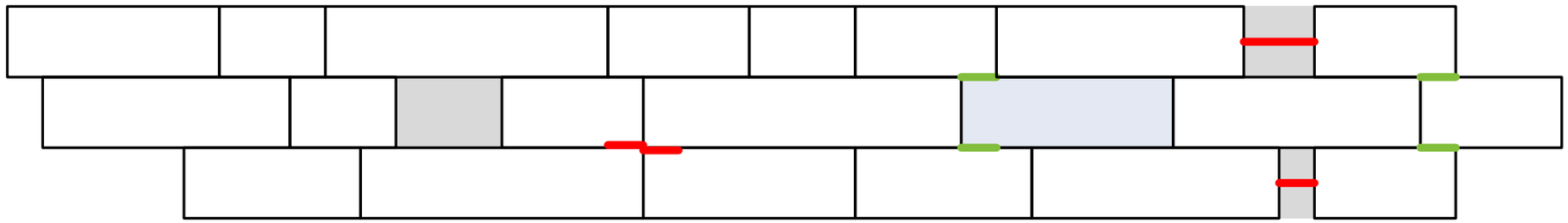
Extension to $L = 3$



Extension to $L = 3$



Extension to $L = 3$



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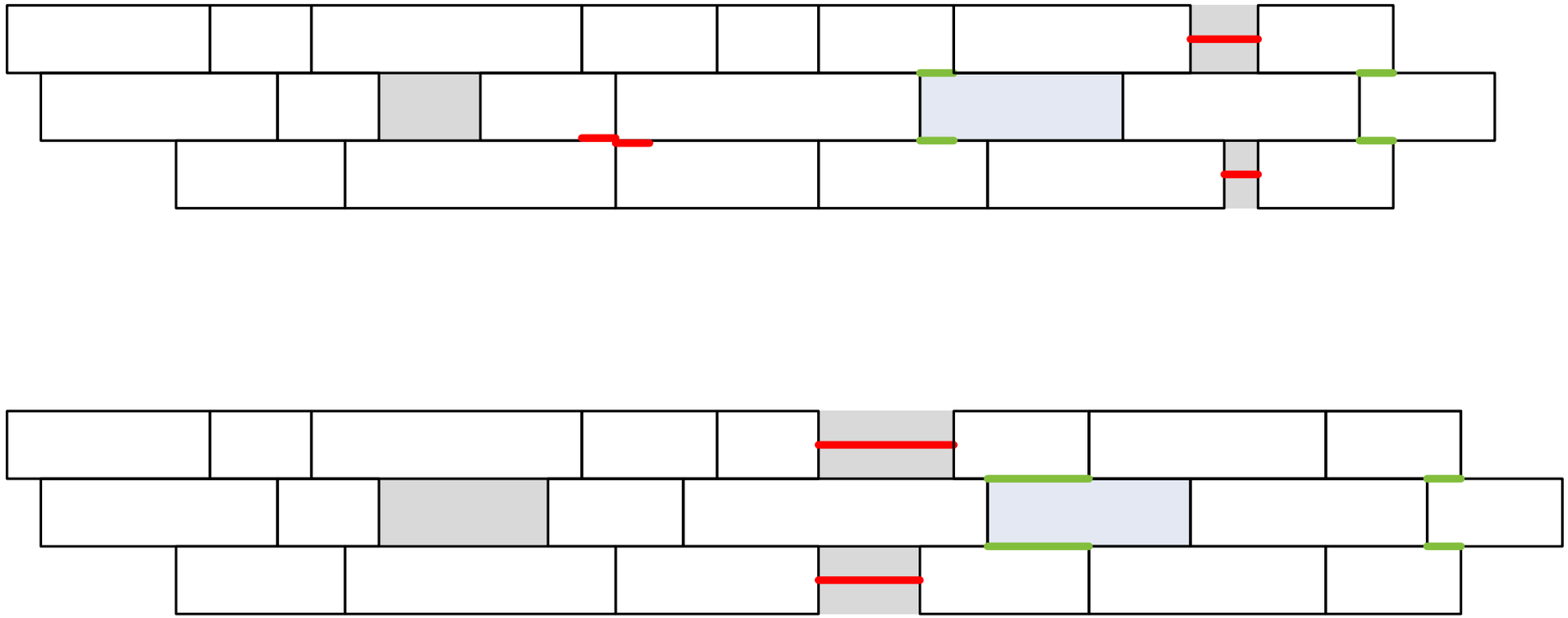

Overview

Contact maximization	Area minimization
Greedy algorithm for $L = 2$	Flow network
Extension to $L = 3$?	
ILP formulation for $L \geq 3$	

Overview

Contact maximization	Area minimization
Greedy algorithm for $L = 2$	Flow network
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ILP formulation for $L \geq 3$	

ILP formulation

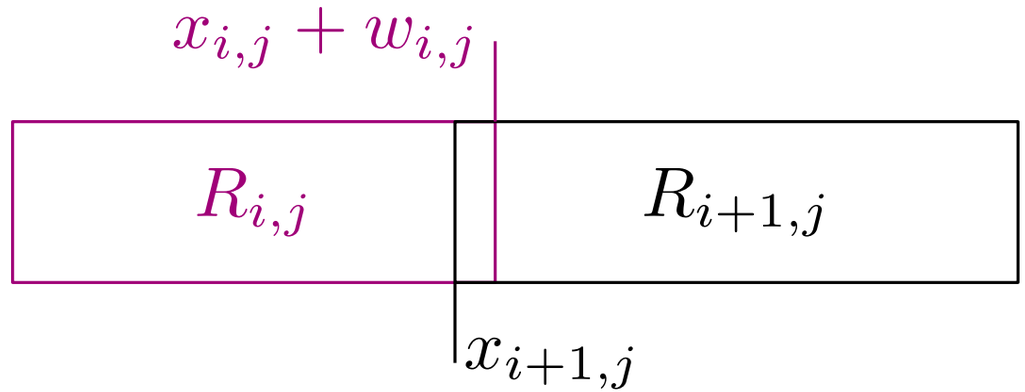
ILP formulation

Hard constraints

ILP formulation

Hard constraints

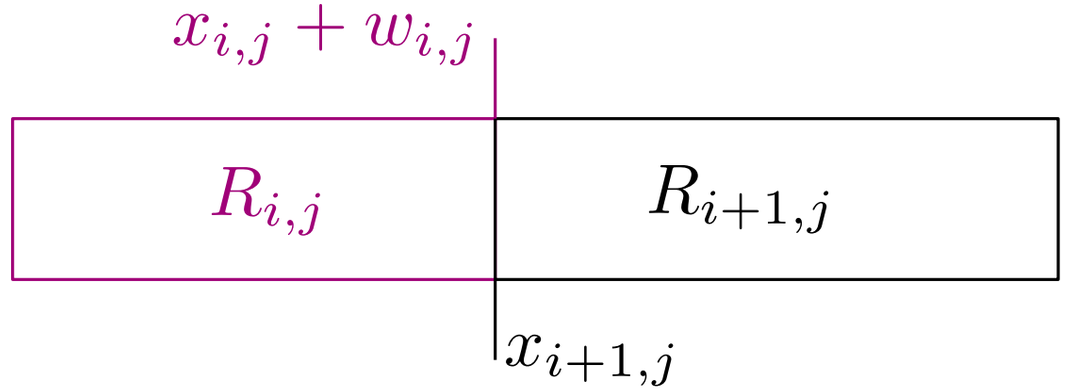
- Prevent overlap



ILP formulation

Hard constraints

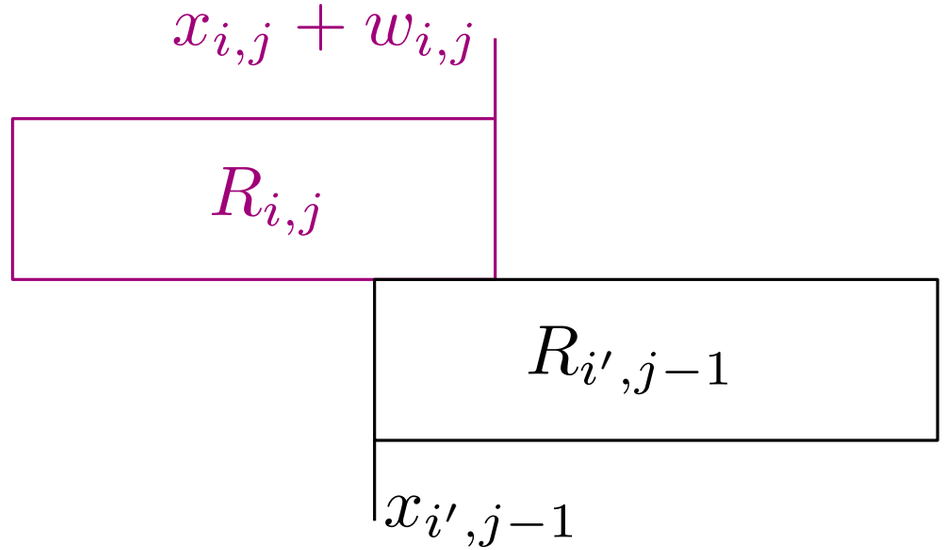
- Prevent overlap



ILP formulation

Hard constraints

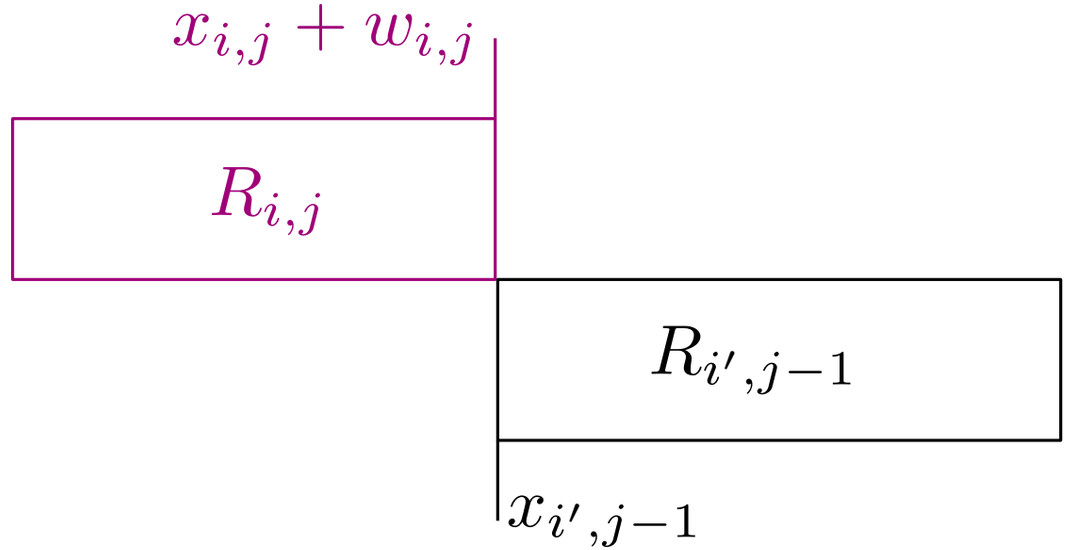
- Prevent overlap
- Prevent false adjacencies



ILP formulation

Hard constraints

- Prevent overlap
- Prevent false adjacencies



ILP formulation

Hard constraints

- Prevent overlap
- Prevent false adjacencies

Soft constraints

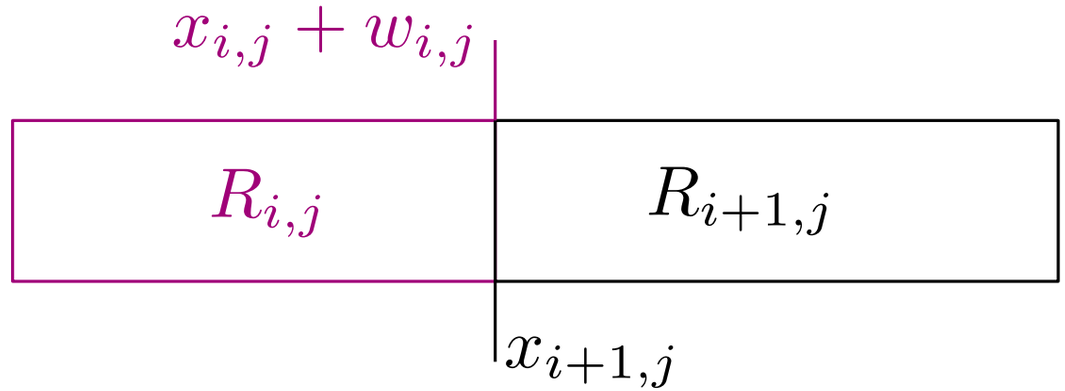
ILP formulation

Hard constraints

- Prevent overlap
- Prevent false adjacencies

Soft constraints

- Increase counter with a failed horizontal contact



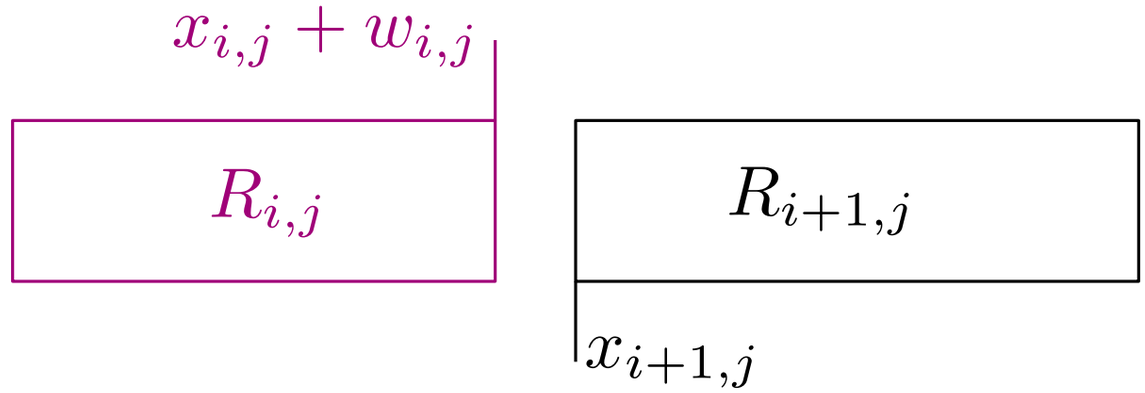
ILP formulation

Hard constraints

- Prevent overlap
- Prevent false adjacencies

Soft constraints

- Increase counter with a failed horizontal contact



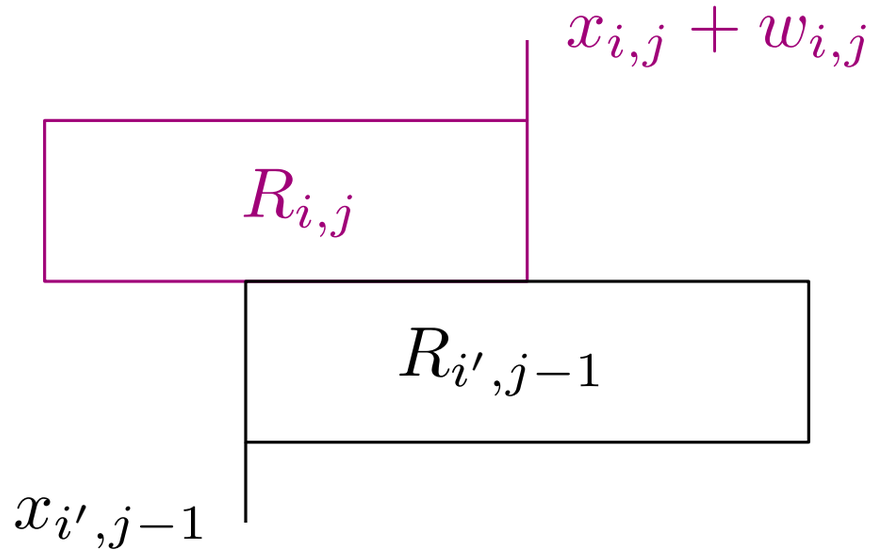
ILP formulation

Hard constraints

- Prevent overlap
- Prevent false adjacencies

Soft constraints

- Increase counter with a failed horizontal contact
- Increase counter with a vertical contact



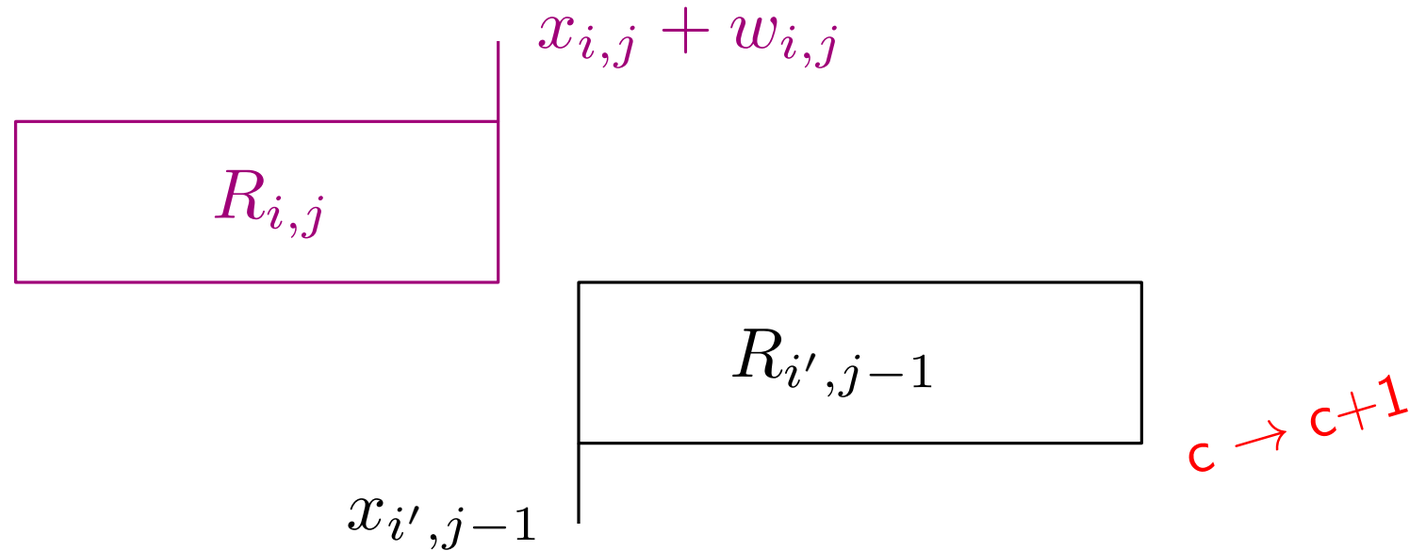
ILP formulation

Hard constraints

- Prevent overlap
- Prevent false adjacencies

Soft constraints

- Increase counter with a failed horizontal contact
- Increase counter with a vertical contact



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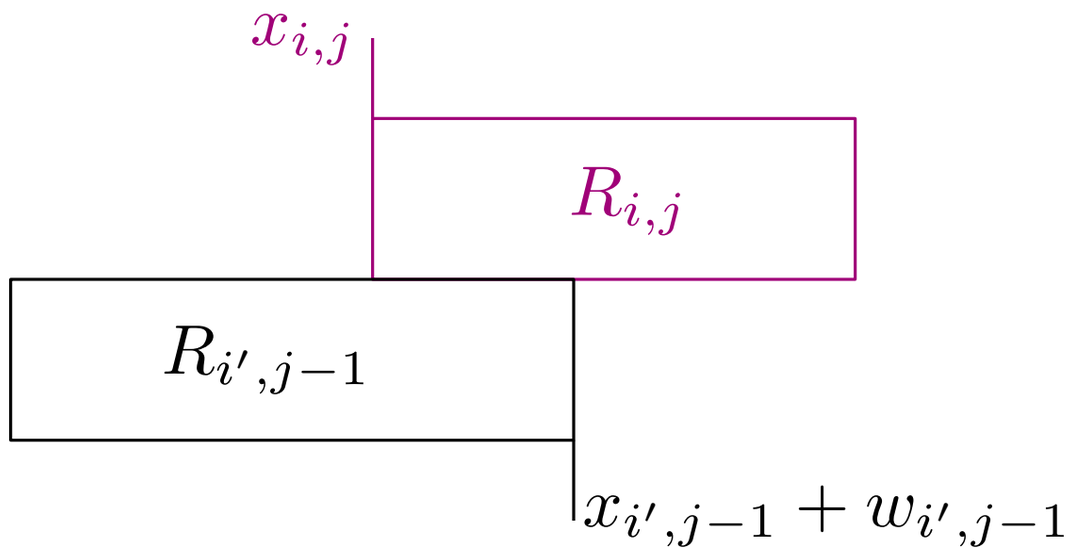

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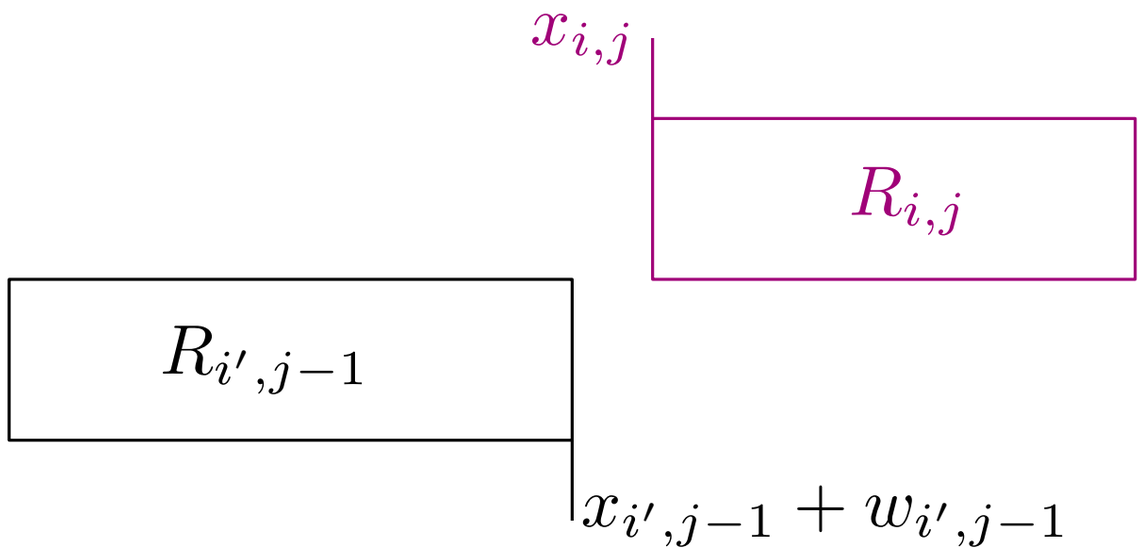

$c \rightarrow c+1$

Overview

Contact maximization	Area minimization
Greedy algorithm for $L = 2$	Flow network
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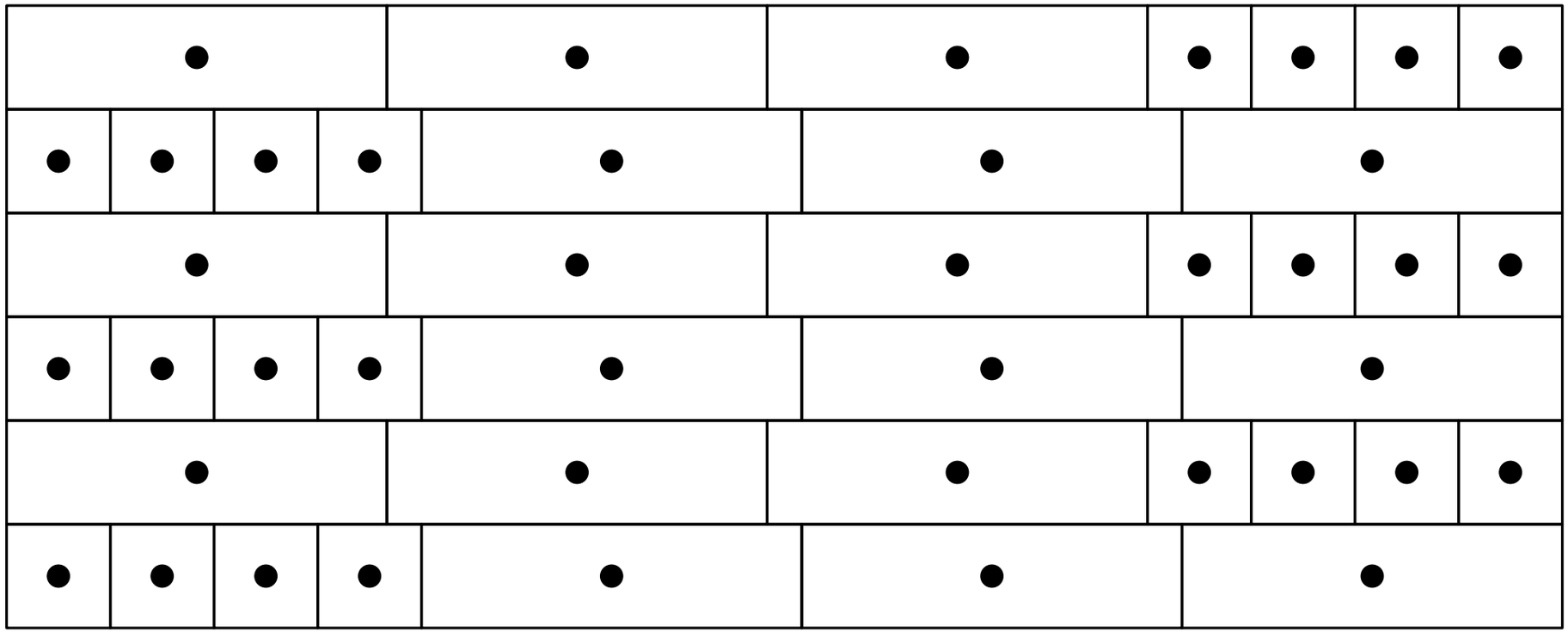
Area minimization

A representation with false adjacencies can have no gap width



Area minimization

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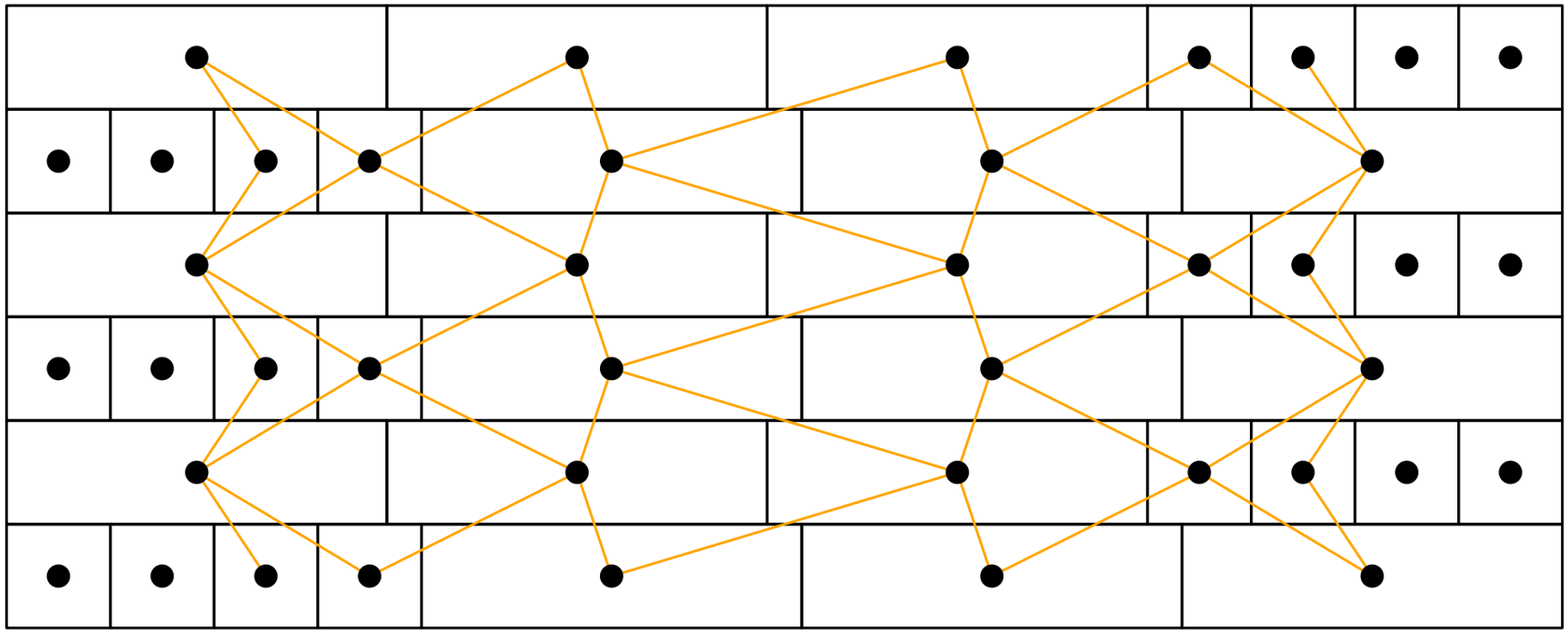
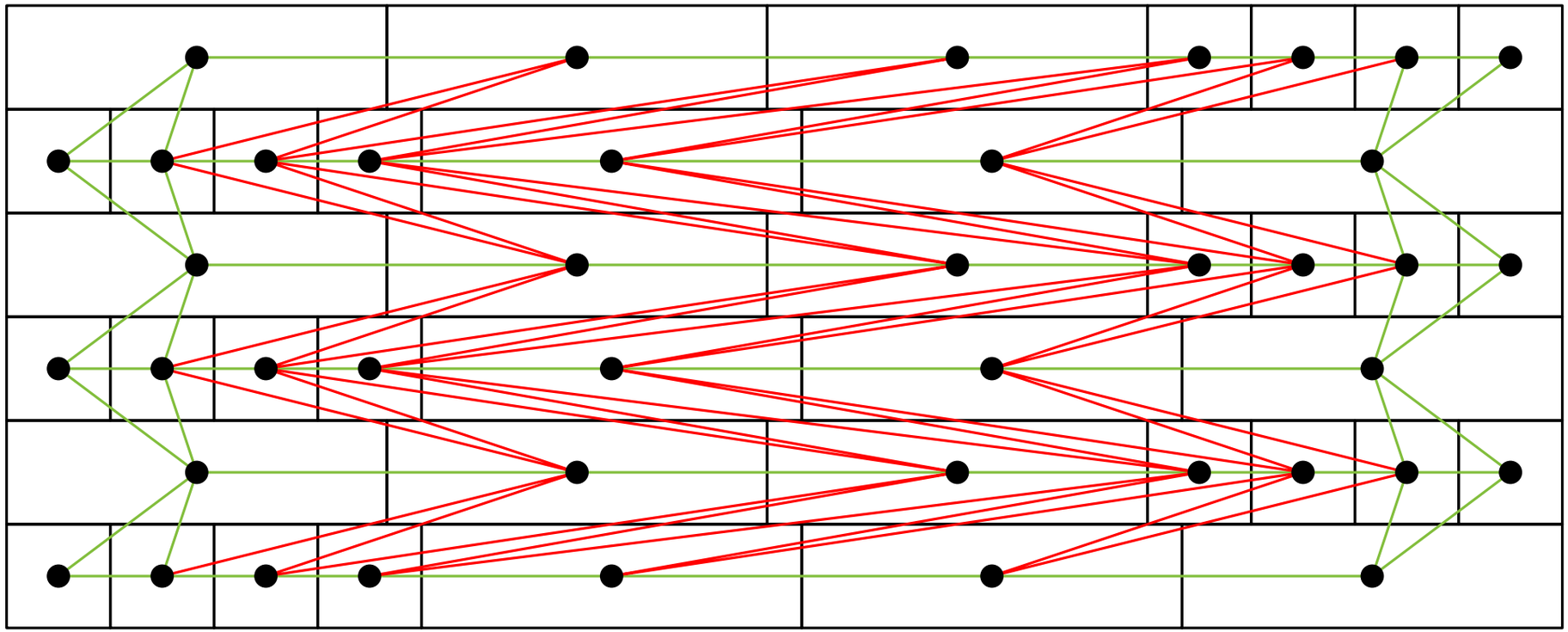


Area minimization

A representation with false adjacencies can have no gap width but lose almost all contacts



Area minimization

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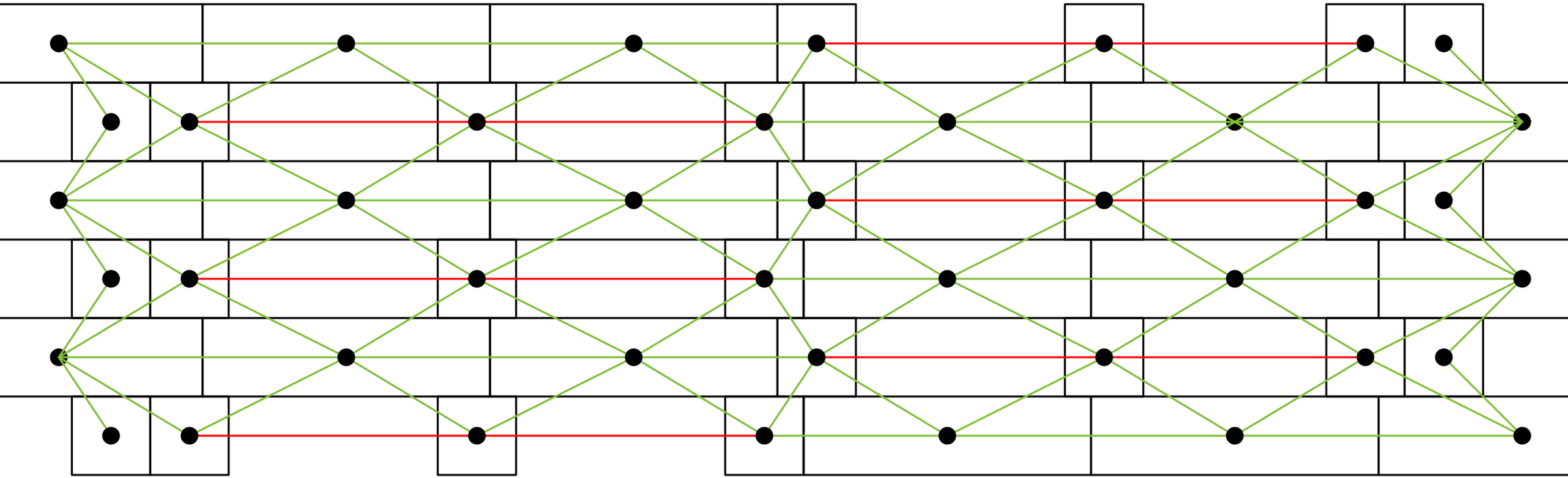
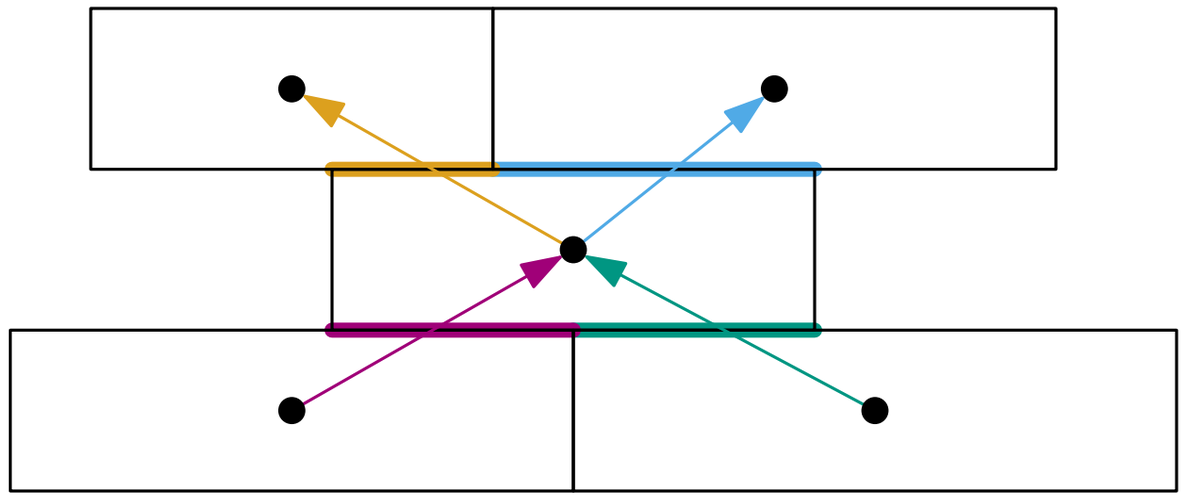


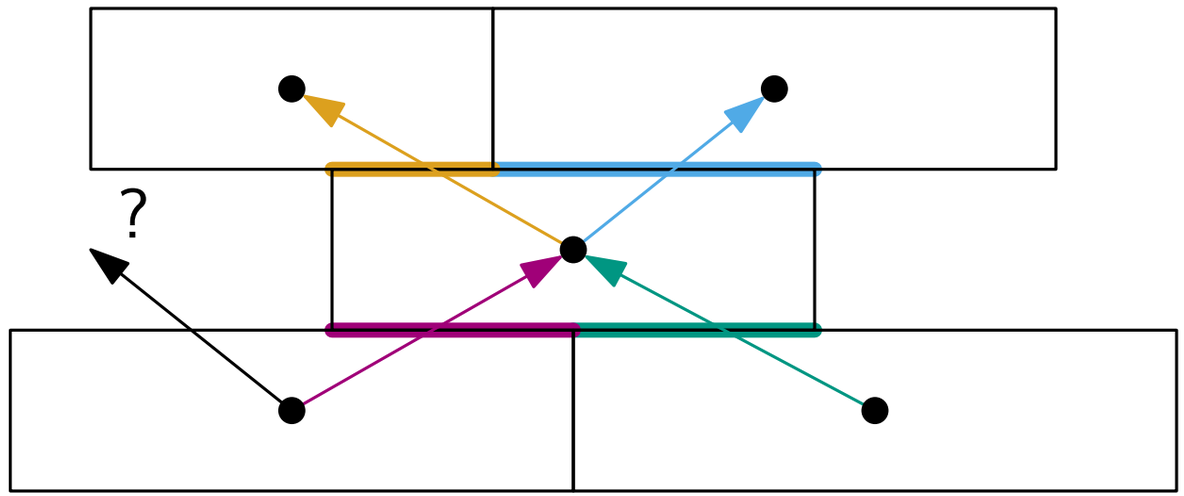
Area minimization

- Flow represents desired contacts between rectangles



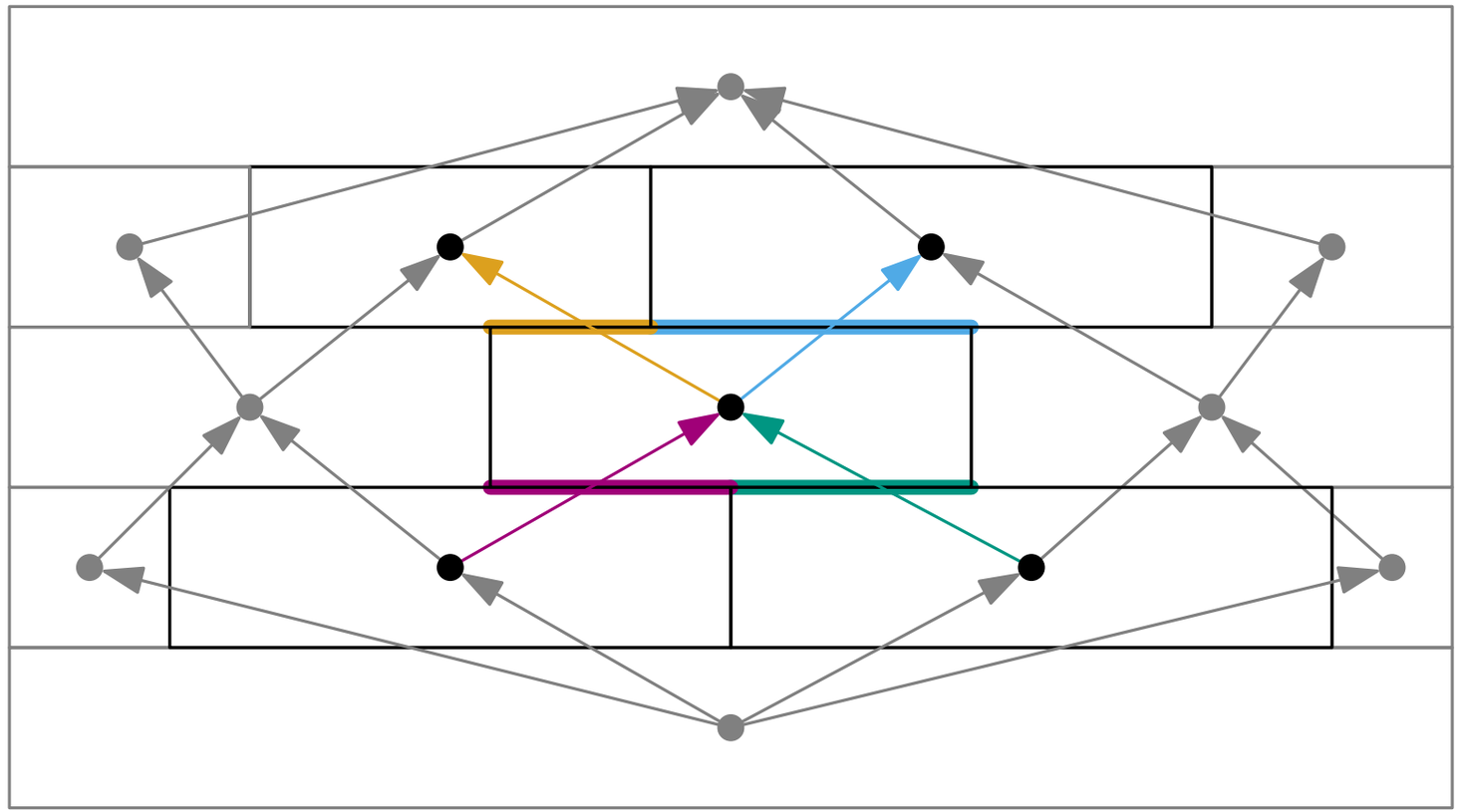
Area minimization

- Flow represents desired contacts between rectangles



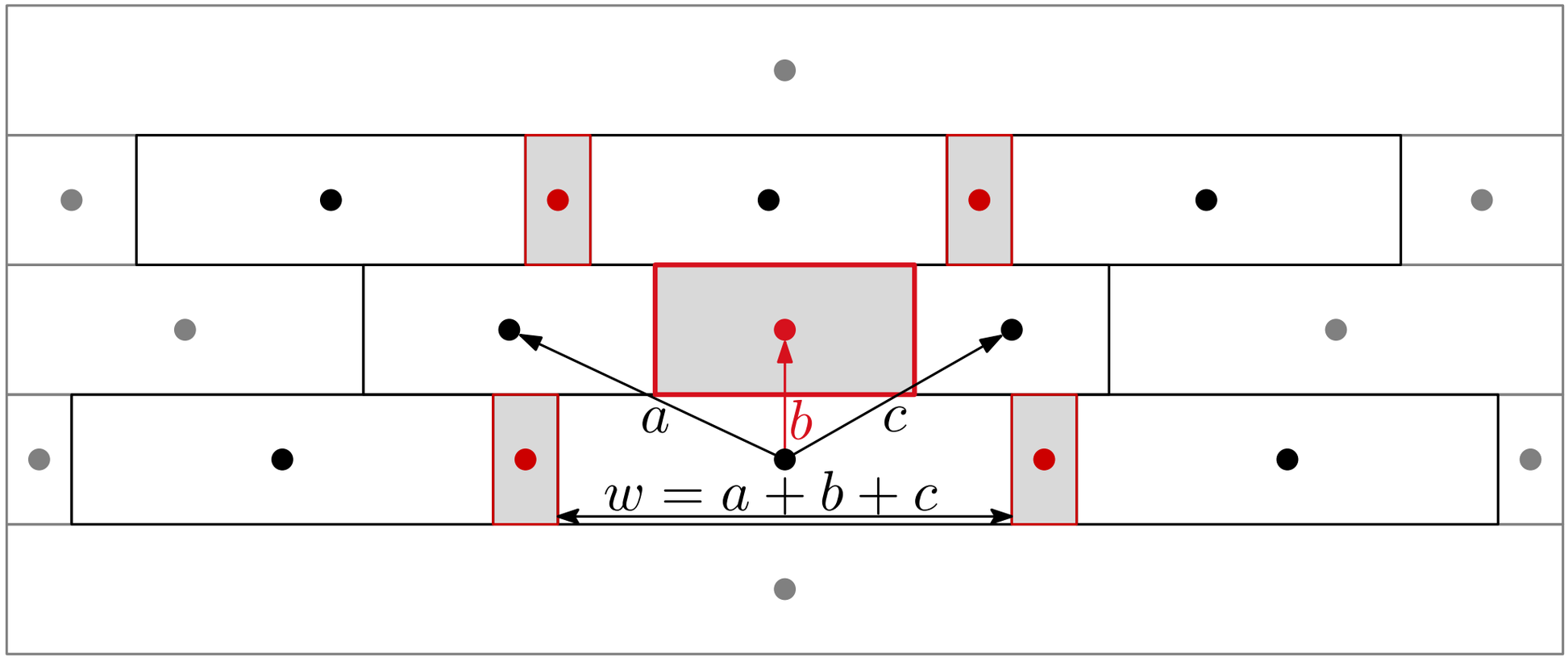
Area minimization

- Flow represents desired contacts between rectangles
- Buffer for excess flow

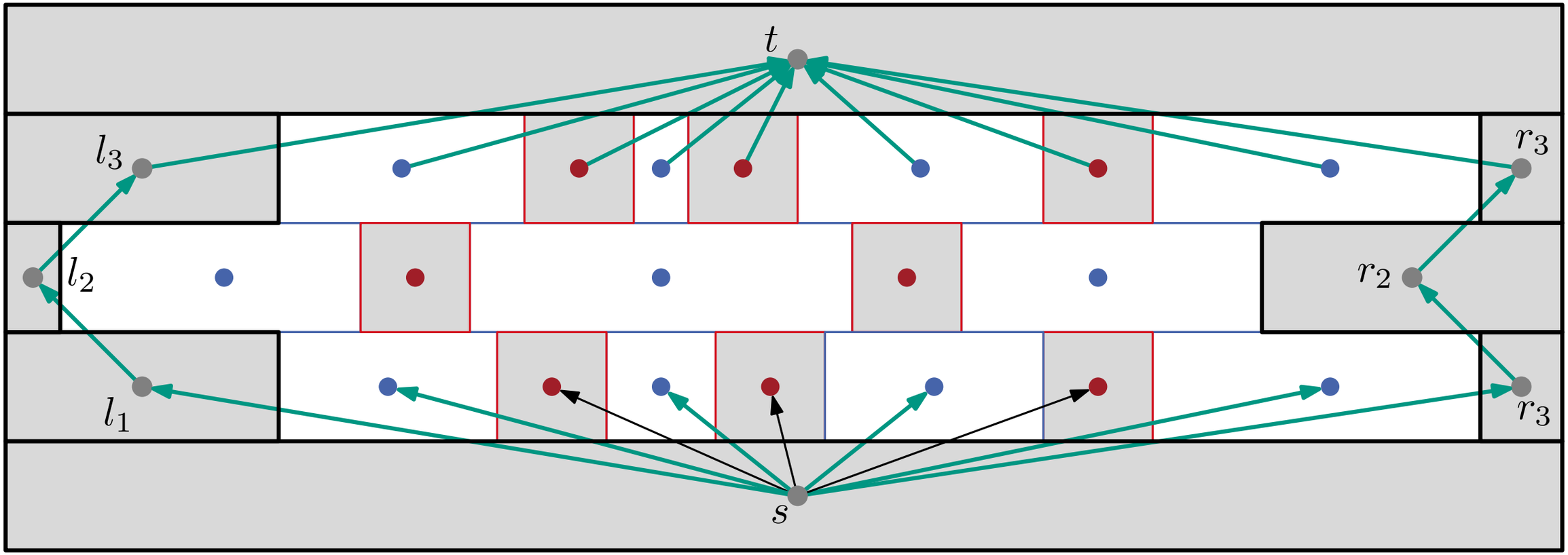


Area minimization

- Flow represents desired contacts between rectangles
- Buffer for excess flow
- Rectangles for flow to gaps



Area minimization



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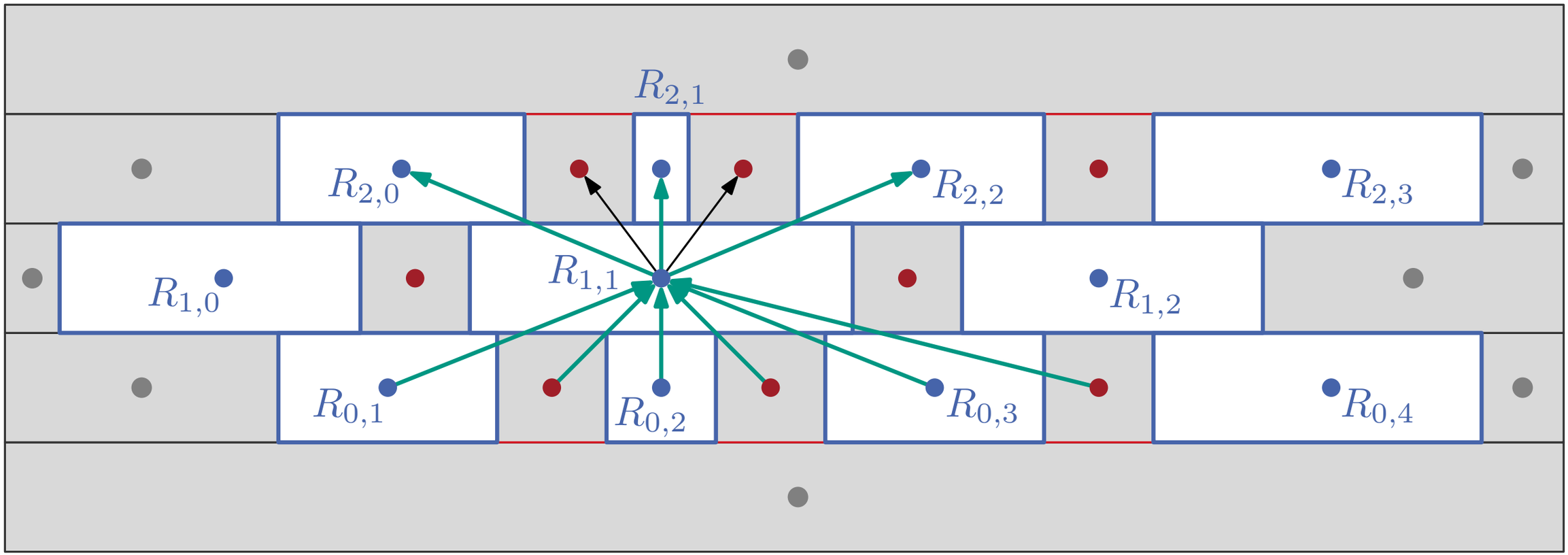

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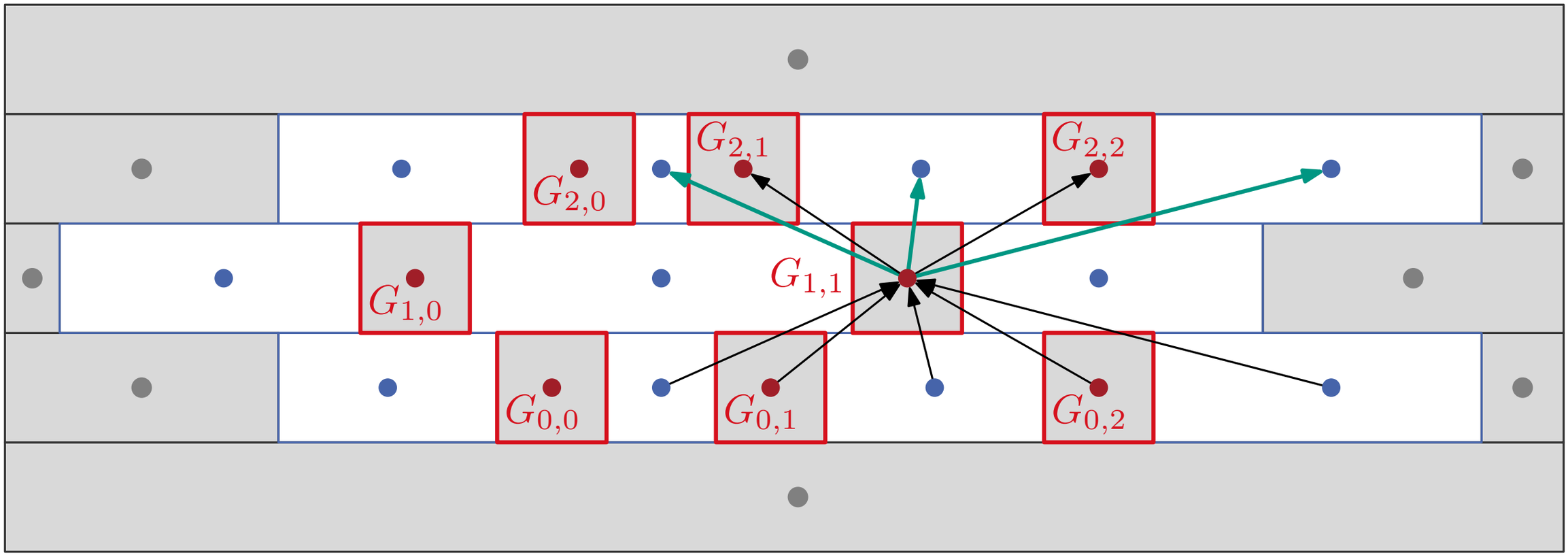
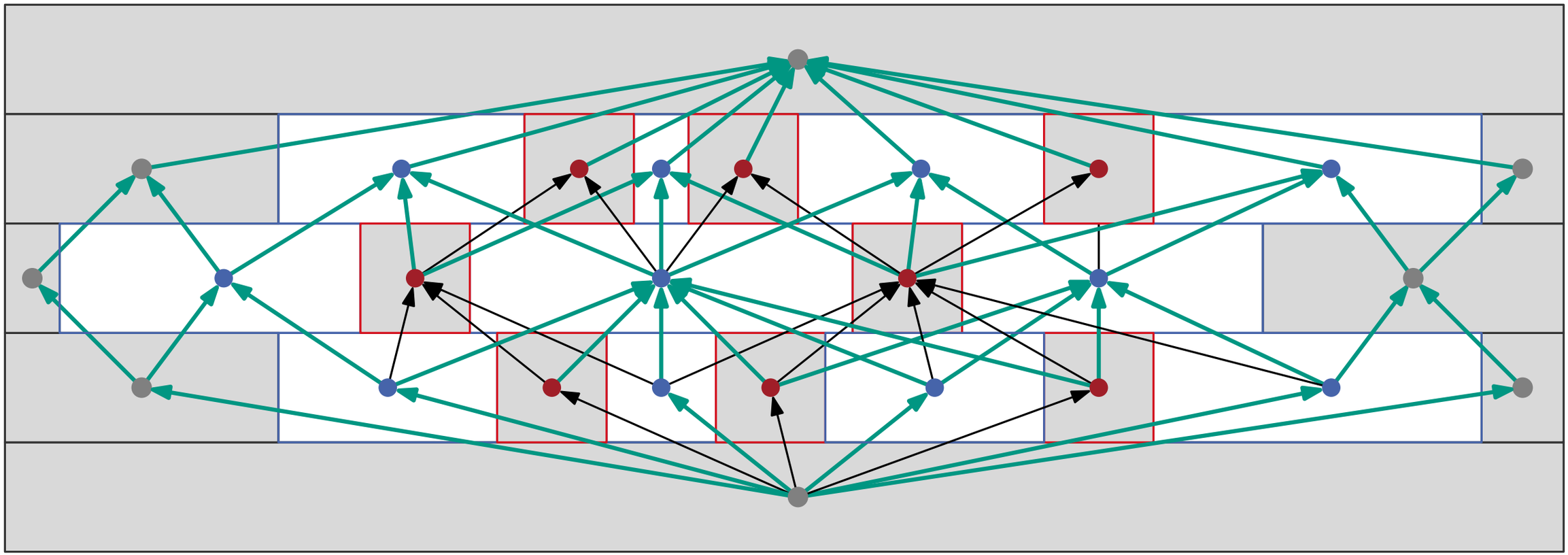


Area minimization



Theorem 2 :

We can minimize the total gap width in polynomial time.

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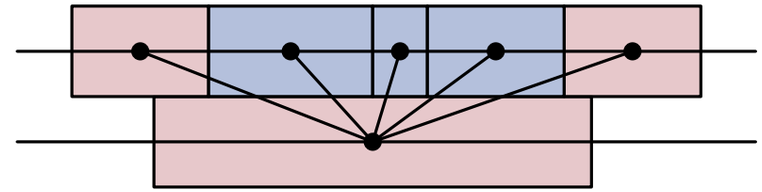
Conclusion

Layered wordles

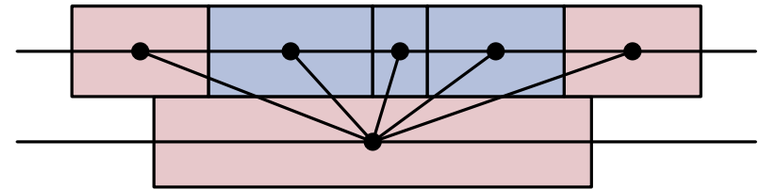
- Contact maximization on $L = 2$

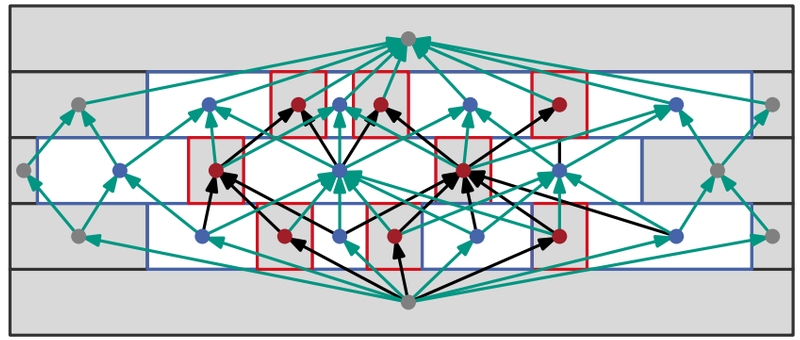


Conclusion

Layered wordles

- Contact maximization on $L = 2$
- Area minimization





Conclusion

Layered wordles

- Contact maximization on $L = 2$
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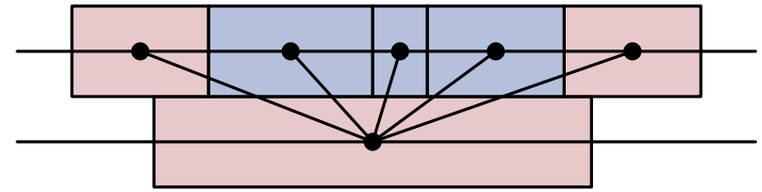
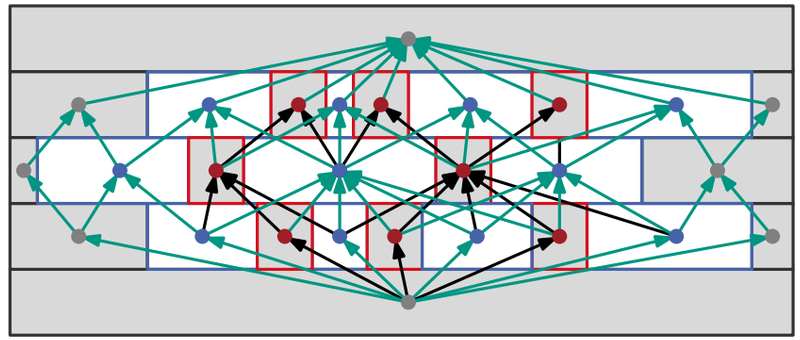
- ILP model





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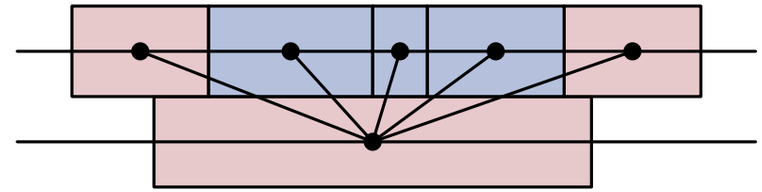

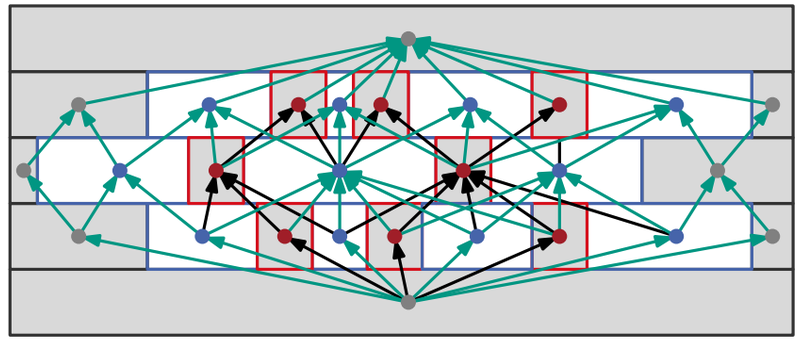

Open problems

- Complexity for $L \geq 3$ problem

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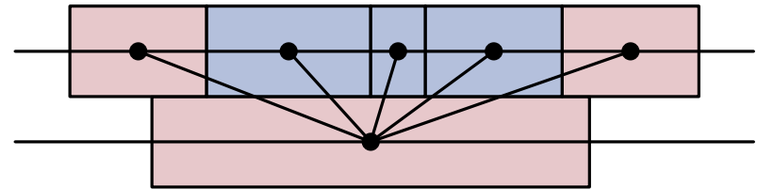

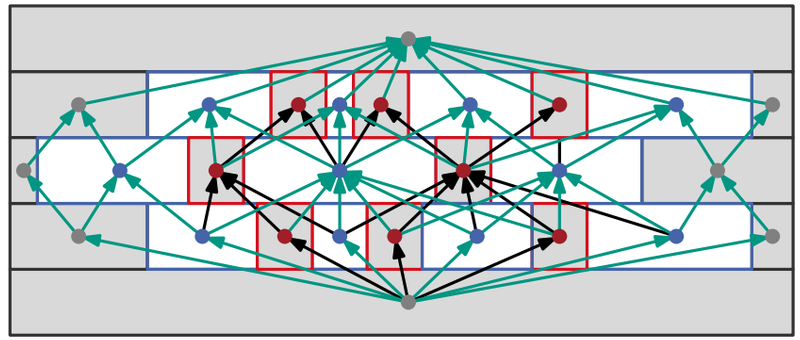

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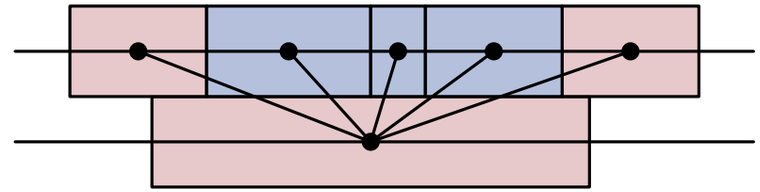
Layered wordles



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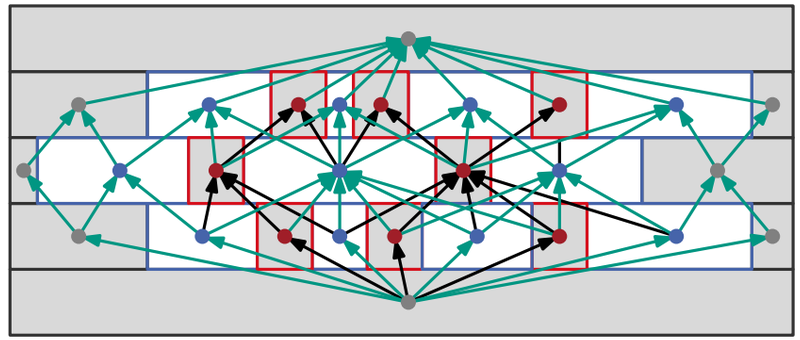

Open problems

- Complexity for $L \geq 3$ problem
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- Generate layered graphs

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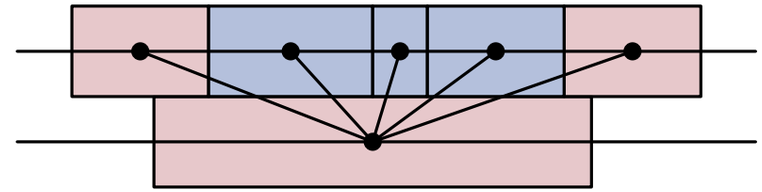

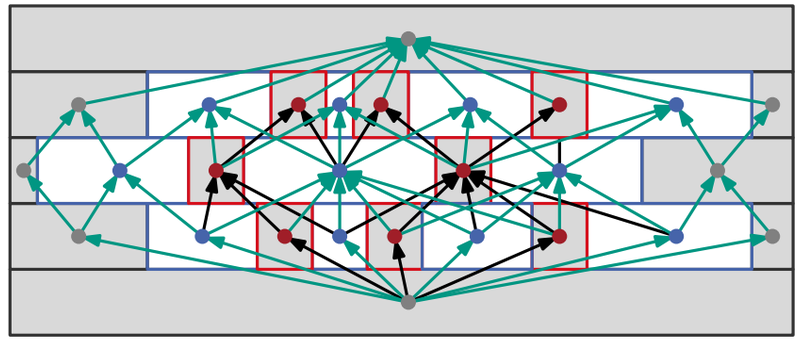

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